Fair Termination for Parameterized Probabilistic Concurrent Systems

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Outline

- Parameterized probabilistic concurrent systems

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- Liveness

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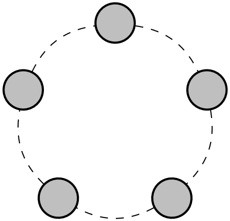
- Liveness
- Fairness
- Regular model checking

Motivating Example

Herman's protocol (merging version)



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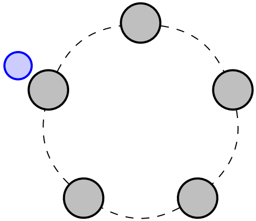

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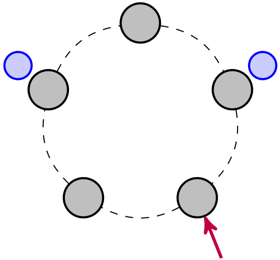

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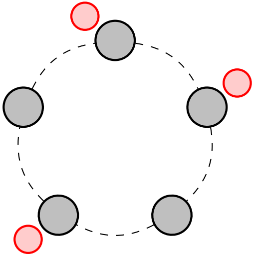

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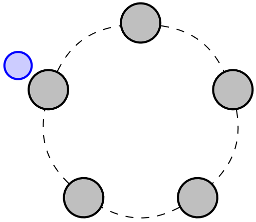

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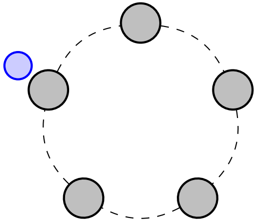

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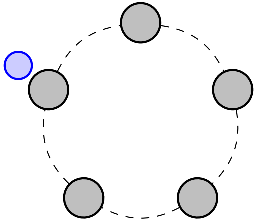

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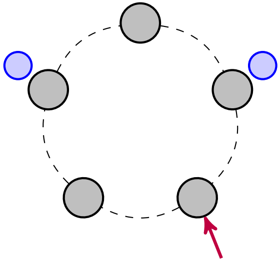

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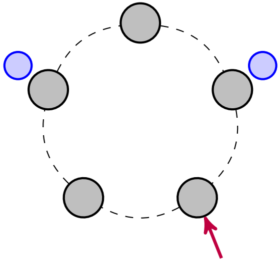

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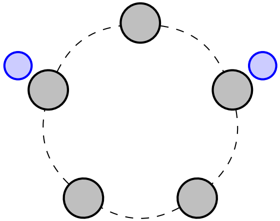

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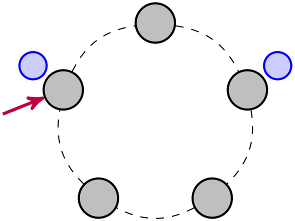

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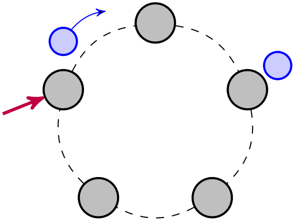

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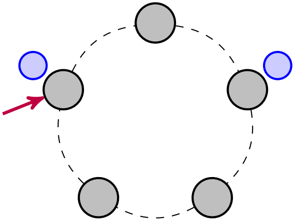

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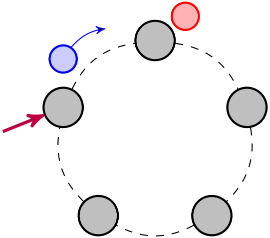

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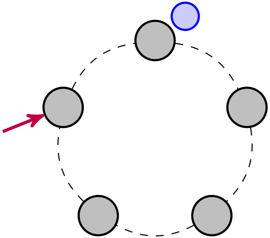

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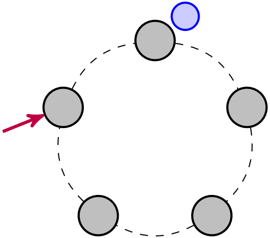

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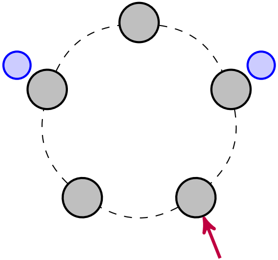

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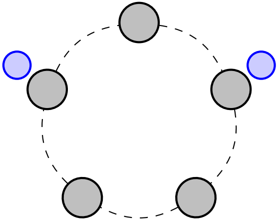

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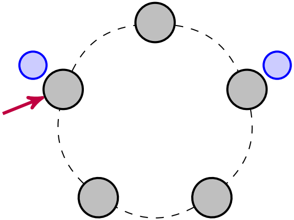

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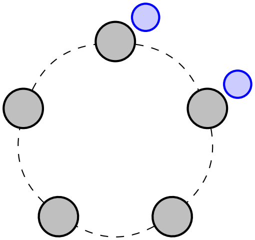

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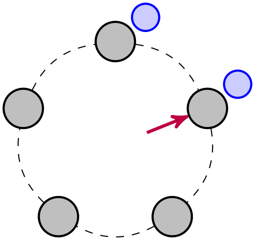

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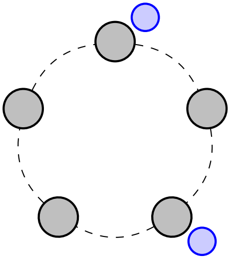

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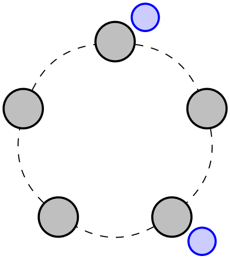

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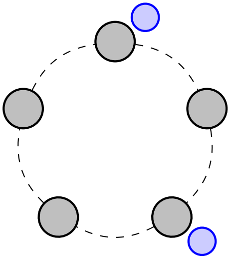

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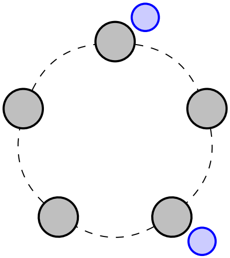

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- We use **finitary** fairness

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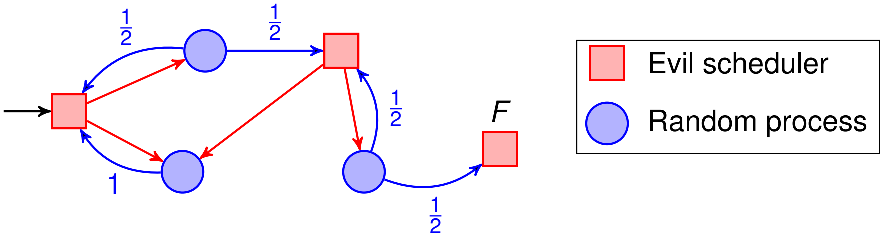

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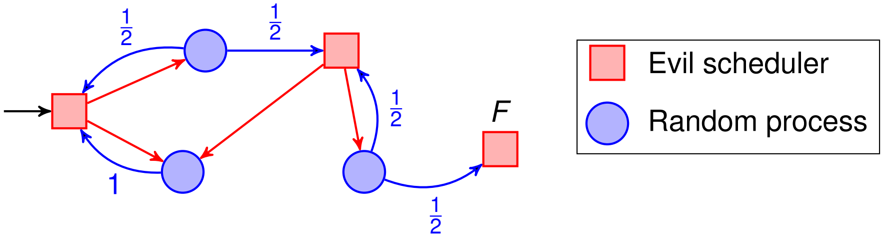

- $\Pr(s_0 \models \Diamond F) \stackrel{?}{=} 1$

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Weakly-finite MDPs:

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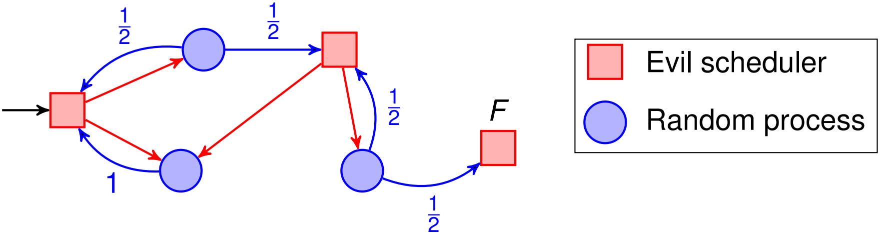

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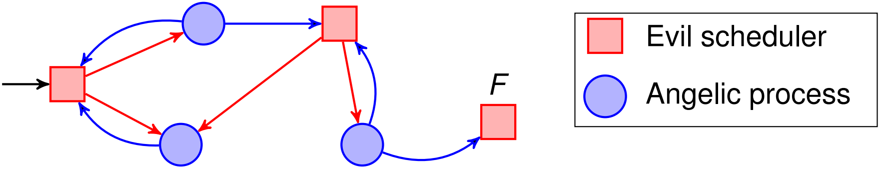

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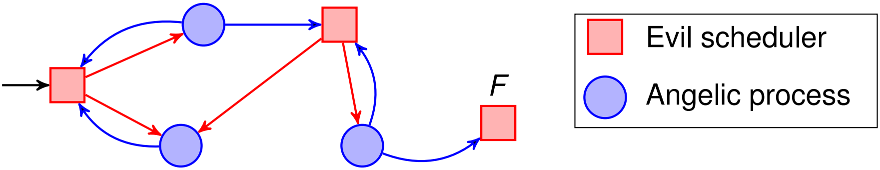

Lemma

$\Pr(s_0 \models \Diamond F) = 1$ *iff Proc. has winning strategy from all* $s \in Reach(s_0)$.

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Regular Model Checking for liveness in param. prob. conc. systems under all schedulers

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- **this talk**: embedding of **fairness** into the system

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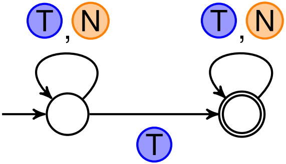

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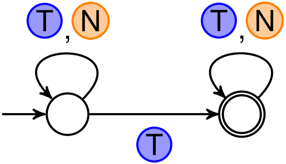
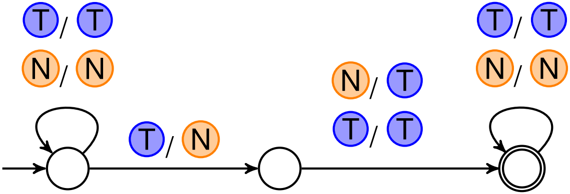


- Transition relation: a (length-preserving) **transducer** τ



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  1. $Start \subseteq Inv$
  2. $\tau_\cup(Inv) \subseteq Inv$
  3. $P_<$ is a strict preorder (i.e., irreflexive, transitive)
  4. For any evil transition from $Inv \setminus Good$ to s_e, there is an angelic transition from s_e that
     - goes to *Inv* and
     - progresses w.r.t. $P_<$

$$\forall x \in Inv \setminus Good, \quad \forall y \in \Sigma^* \setminus Good :$$
$$(x \rightarrow_{\tau_1} y) \Rightarrow (\exists z \in Inv : (y \rightarrow_{\tau_2} z \wedge z <_P x))$$

Finitary Fairness — [Alur & Henzinger'98]

k-Fairness

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- *intuition*: binds the scope of $\Box$ and $\Diamond$ operators to k steps.

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Finitary fairness: if k-fair for some k

Encoding Finitary Fairness into RMC

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- Generalized to arbitrary weak and strong fairness

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Example: Herman's protocol:

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  N 1 1 0 | T 1 1 1 | **T 1 1 0** | N 1 0 0
- process player decrements/resets counters

  N 1 0 0 | T 1 1 0 | **T 1 1 1** | N 0 0 0

Encoding Finitary Fairness into RMC

Theorem

Let S be a regular representation of an MDP with finitary fairness constraints C. The presented transformation yields a regular representation of an MDP S_F (without fairness constraints) such that (if C are realizable)

$$\Pr(\mathit{Start} \models \Diamond \mathit{Good}) = 1 \qquad \textit{iff} \qquad \Pr(\mathit{Start}_F \models \Diamond \mathit{Good}_F) = 1$$

Case Studies: Population Protocols

Moran process

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- a model of genetic drift

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  - (similar for (B))

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  - ... (B) [(A)] ... ⇝ ... (A) [(A)] ...
  - (similar for [(B)])
- goal: $(A)^*$ or $(B)^*$

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  - $\ldots$ Ⓑ Ⓐ $\ldots \rightsquigarrow \ldots$ Ⓐ Ⓐ $\ldots$
  - (similar for Ⓑ)
- goal: Ⓐ* or Ⓑ*
- **Cell cycle switch** — similar, but has an intermediate state

Case Studies: Population Protocols

Clustering

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  - … (B)(A) … ⇝ … (A)(B) …
  - (similar for (B))
- goal: $(A)^*(B)^*$ or $(B)^*(A)^*$

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- a population of **agents**
- every agent has one currency: Dollars or Euros
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- Encoding implemented in FAIRYTAIL

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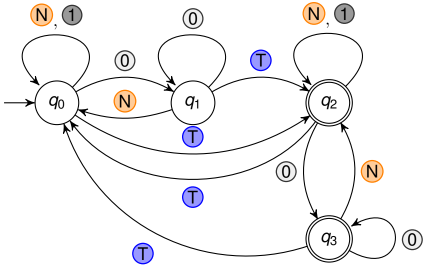
  - SYNTHESISE: use a SAT solver (Sat4j) to obtain a candidate
  - VERIFY: check the candidate is OK/refine SAT formula

Evaluation

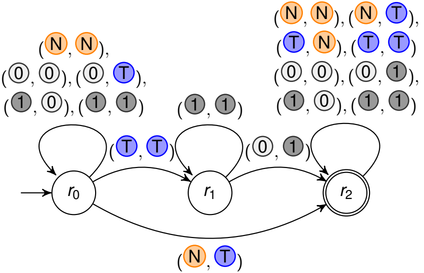
Table: Results of experiments (timeout = 10 hours).

Case study	Time
Herman's protocol (merge, line)	3.64 s
Herman's protocol (annih., line)	4.33 s
Herman's protocol (merge, ring)	4.31 s
Herman's protocol (annih., ring)	4.61 s
Moran process (2 types, line)	2 m 48 s
Moran process (3 types, line)	56 m 14 s
Cell cycle switch (1 types, line)	43.94 s
Cell cycle switch (2 types, line)	9 h 46 m
Clustering (2 types, line)	10 m 30 s
Clustering (3 types, line)	T/O
Coin game ($k = 3$, clique)	1 m 0 s

Solution to Herman's protocol (merge, ring)



Inv



$P_<$

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