A Uniform Framework for Handling Position Constraints in String Solving

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String Solving

- Satisfiability of formulas over string constraints such as:

$$\underbrace{x = yz}_{\text{equations}} \land \overbrace{yz \neq ua}^{\text{disequalities}} \land \underbrace{x \in (ab)^* a^+ (b|c)}_{\text{regular constraints}} \land \overbrace{|xy| = 2|uv| + 1}^{\text{length constraints}} \land \underbrace{\neg contains(uxz, zbcx)}_{\text{more complex operations}}$$

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  - scripting languages rely heavily on strings
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- e.g.,

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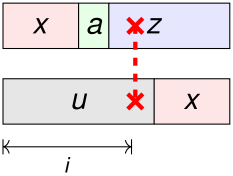

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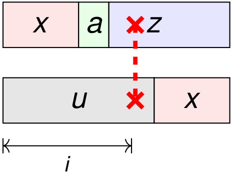

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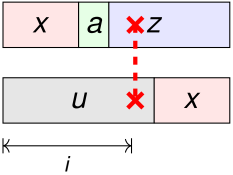

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$$\neg contains(xya, zyw) \quad \Leftrightarrow \quad \forall k \colon 0 \leq k \leq |zyw| - |xya| \Rightarrow \exists i \colon (xya)[i] \neq (zyw)[i + k]$$

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3. Solve $PF(\mathcal{A}_{tag})$ by an off-the-shelf LIA solver

Tag Automaton

Tag automaton over set of tags $\mathbb{T}$:

- extension of finite automaton
- $\mathcal{A}_{tag} = (Q, \Delta, I, F)$
  - Q: (finite) set of states
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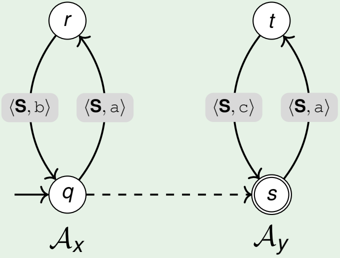

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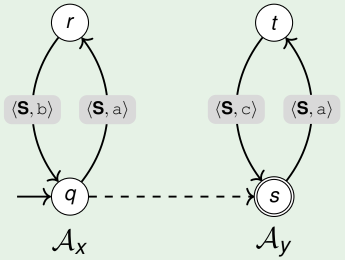

Parikh formula $PF(\mathcal{A}_{tag})$:

- a **linear integer arithmetic** (LIA) formula over variables $\#\mathbb{T} = \{\#t \mid t \in \mathbb{T}\}$
- assignments $\{\#t \mapsto n_t \mid t \in \mathbb{T}, n_t \in \mathbb{N}\}$ (simplified)
- $m \models PF(\mathcal{A}_{tag})$ iff there is an accepting run in $\mathcal{A}_{tag}$ s.t. $m(\#t)$ is the number of occurrences of a tag in a word accepted by $\mathcal{A}_{tag}$.
- e.g., if $ababacac \in L(\mathcal{A}_{tag})$ then $\{\#\langle \mathbf{S}, a\rangle = 4, \#\langle \mathbf{S}, b\rangle = 2, \#\langle \mathbf{S}, c\rangle = 2\} \models PF(\mathcal{A}_{tag})$

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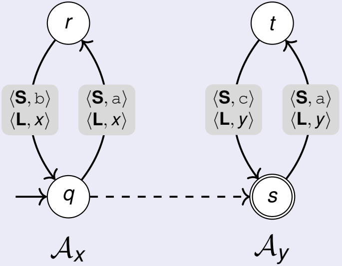

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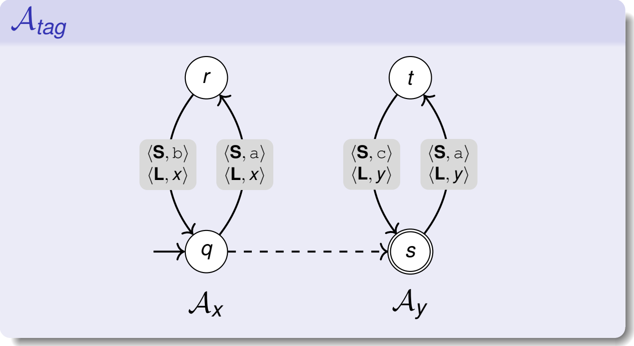

$$\varphi\colon PF(\mathcal{A}_{tag}) \quad \wedge \quad 2 \cdot \#\langle \mathbf{L}, x\rangle = 3 \cdot \#\langle \mathbf{L}, y\rangle + 2$$

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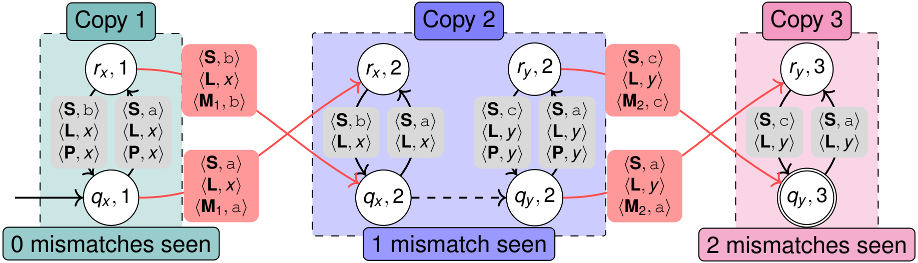

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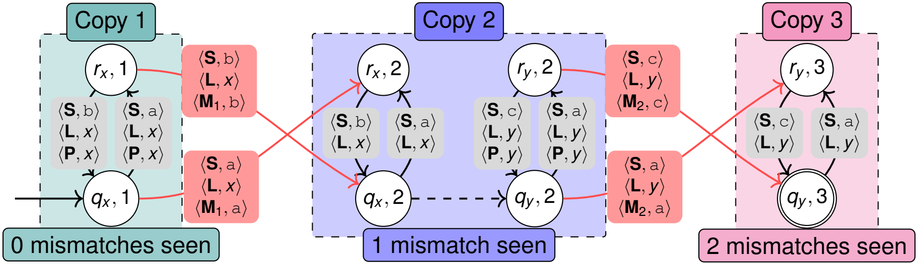

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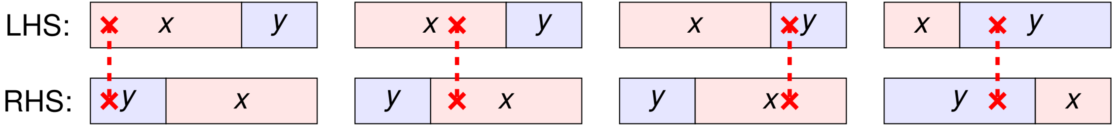

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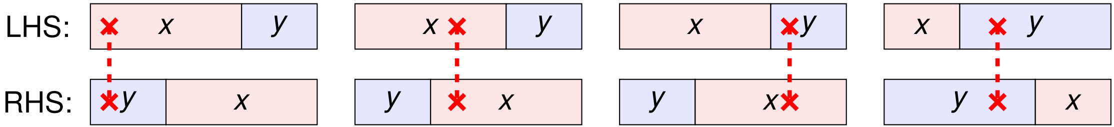

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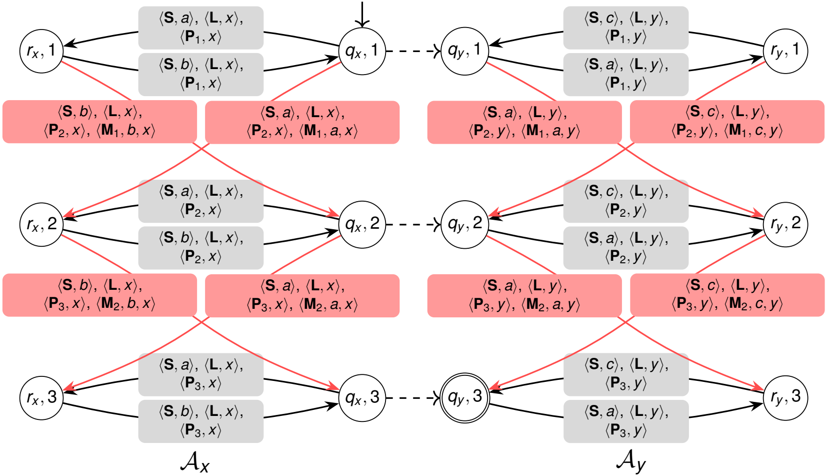

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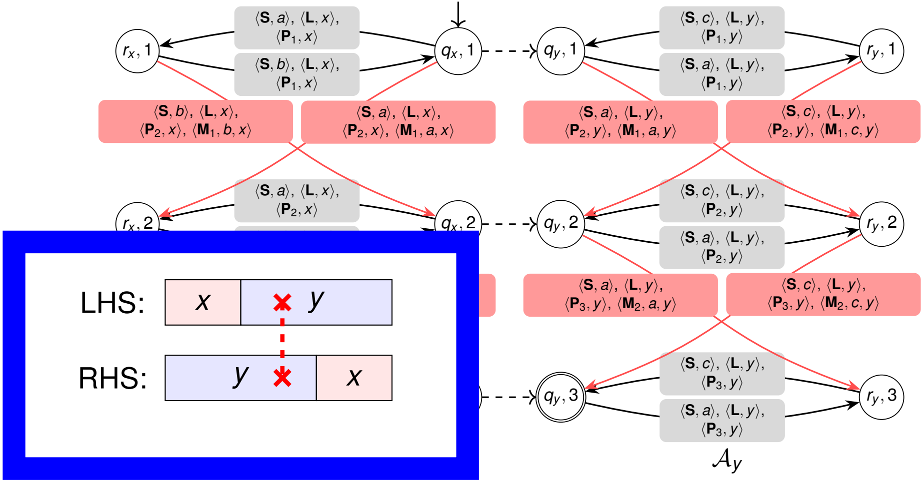

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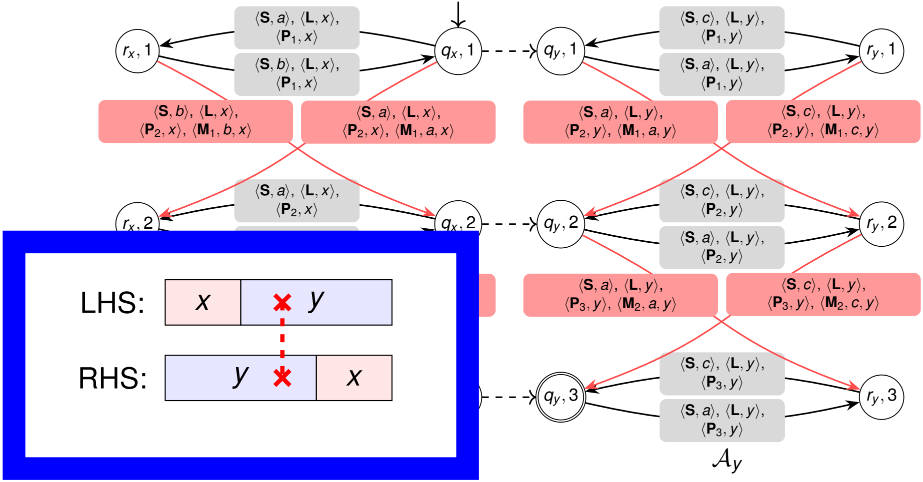

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Other constraints

- $\neg \mathit{prefixof}$, $\neg \mathit{suffixof}$, $\mathit{str.at}$, $\neg \mathit{str.at}$: similar technique
- $\neg \mathit{contains}$: $\ldots$

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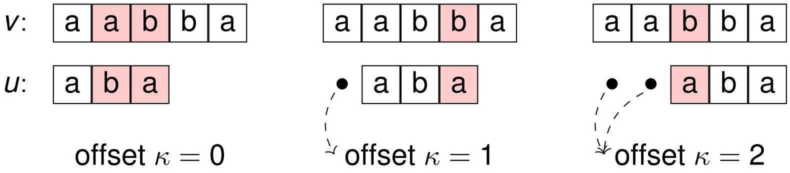

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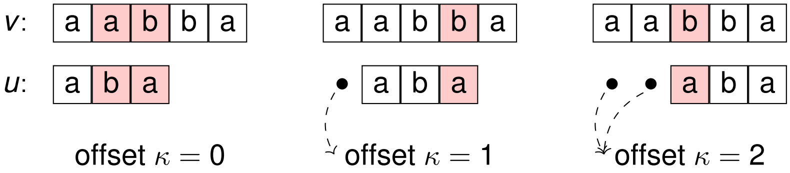

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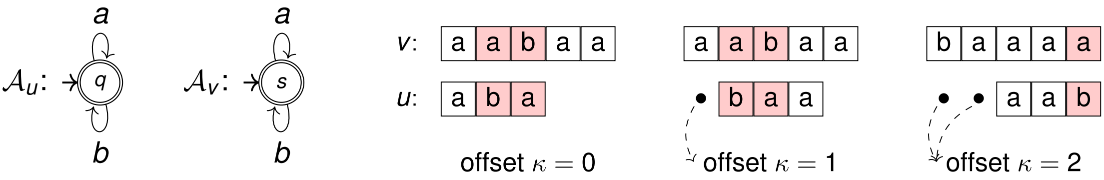

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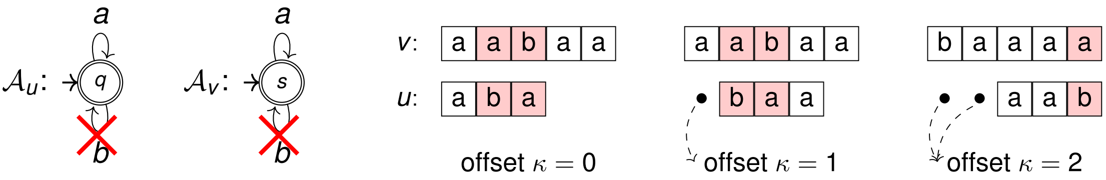

- restriction to flat regular constraints
  - $\forall\exists$ LIA formula
- details in the paper!

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- REG with multiple position constraints, lengths, and chain-free word equations
  - decidable (in ELEMENTARY)
  - efficient in practice!

Experimental Evaluation

- implemented in **Z3-NOODLER-POS** — extension of Z3-NOODLER
- compared to
  - Z3-NOODLER
  - CVC5
  - Z3
  - OSTRICH
- benchmarks:
  - **symbolic execution**[1]: using Python PyCT symbolic executor
    - biopython (77,222): bioinformatics Python tools
    - django (52,643): Django Python web app
    - thefuck (19,872): Python command mistake correction tool
  - **hand-crafted**:
    - position-hard (550): difficult small formulae with $\neq$ and $\neg$*contains*

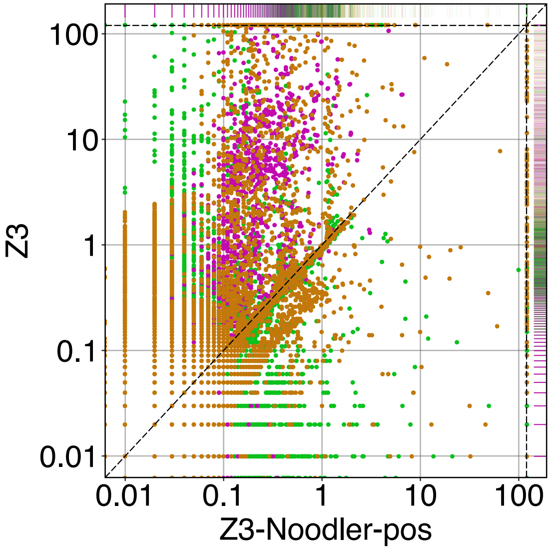
[1] Abdulla et al. "Solving not-substring constraint with flat abstraction". In: *APLAS'21*.

Experimental Evaluation

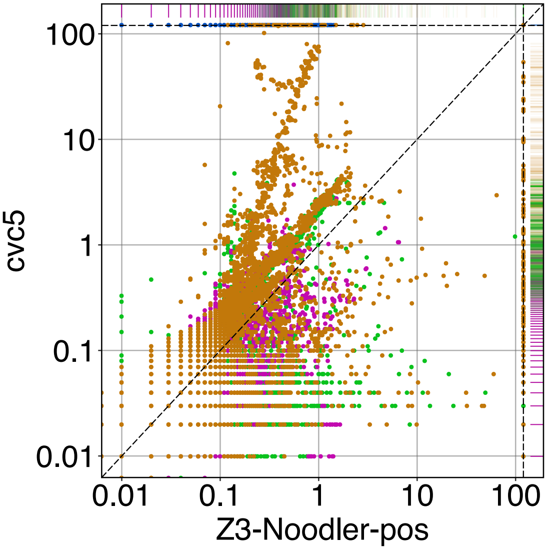
	biopython (77,222)		django (52,643)		thefuck (19,872)		position-hard (550)		All (150,287)	
	Unsolved	TimeAll	Unsolved	TimeAll	Unsolved	TimeAll	Unsolved	TimeAll	Unsolved	TimeAll
Z3-NOODLER-POS	171	24,010	39	8,005	0	**665**	0	**124**	210	**32,804**
Z3-NOODLER	507	64,385	145	20,873	376	45,757	480	59,512	1,508	190,527
CVC5	69	**21,114**	0	**4,515**	0	690	550	66,000	619	92,319
Z3	1,047	141,301	502	67,741	47	15,097	550	66,000	2,146	290,139
OSTRICH	2,986	1,108,306	4,404	1,507,806	967	236,192	550	66,000	8,907	2,918,304

- **Unsolved**: out of resources (timeout: 120 s) or Unk
- **TimeAll**: time-of-solved + (timeout $*$ #-of-failed-instances)

Comparison with Z3 and cvc5



(a) vs. Z3



(b) vs. cvc5

Conclusion and Future Work

Takeaway:

- regular + position constraints $\rightsquigarrow$
  - $\rightsquigarrow$ tag automaton $\rightsquigarrow$
  - $\rightsquigarrow$ Parikh formula $\rightsquigarrow$
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- non-flat $\neg$*contains*
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  - backreferences, bounded repetition, ...

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Thank you!

Word Equations: $uxa = buw$

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- word equations
  - $\rightsquigarrow$ can be transformed to regular constraints
    - basic algorithm of main automata-based solvers (NORN, OSTRICH, Z3-NOODLER, ...)
    - obtain the so-called monadic decomposition
    - incomplete, but mostly works in practice
- non-negated predicates: can be encoded as word equations
  - e.g., $\mathit{prefixof}(x, y) \quad \Leftrightarrow \quad x = yz$

Standard Approach for Position Constraints

Standard approach for handling position constraints:

- encode to word equations
- disequalities: $x \neq y$:

$$\bigvee_{\substack{c_1, c_2 \in \Sigma \\ c_1 \neq c_2}} \alpha c_1 \beta = xyz \land \alpha c_2 \beta' = uawx$$

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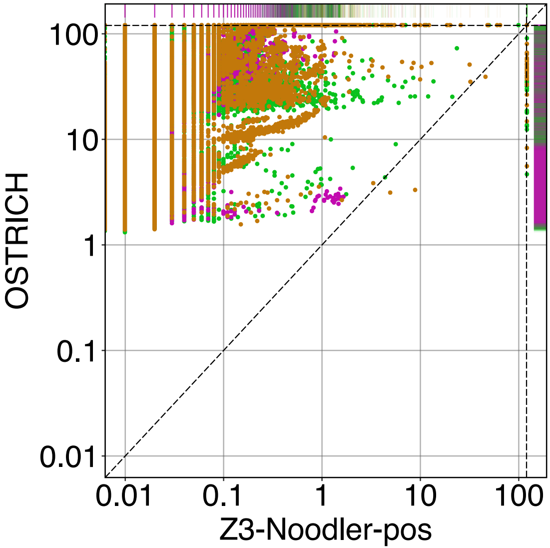
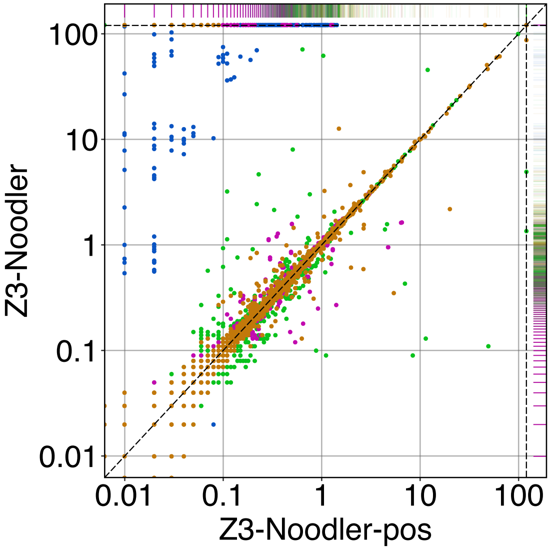
- solving word equations is in PSPACE (but often solved by algorithms)
- breaks chain-freeness
- $\neg$*contains*: no standard way
  - can be encoded in the $\forall\exists$ fragment of string constraints (undecidable)
  - $\neg contains(x, y)$

Comparison with OSTRICH and Z3-NOODLER



(a) Z3-NOODLER-POS vs. OSTRICH



(b) Z3-NOODLER-POS vs. Z3-NOODLER