AN **AUTOMATA-BASED** APPROACH FOR SCALABLE VERIFICATION OF **QUANTUM CIRCUITS**

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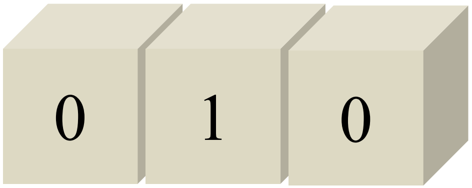
- **Do not be scared!**
  - We assume no background in quantum and minimal background in automata theory.

Outline

- **Minimal Quantum Background and Motivation**
- Quantum Circuit Verification
- Evaluation

A 3-Bit Classical State

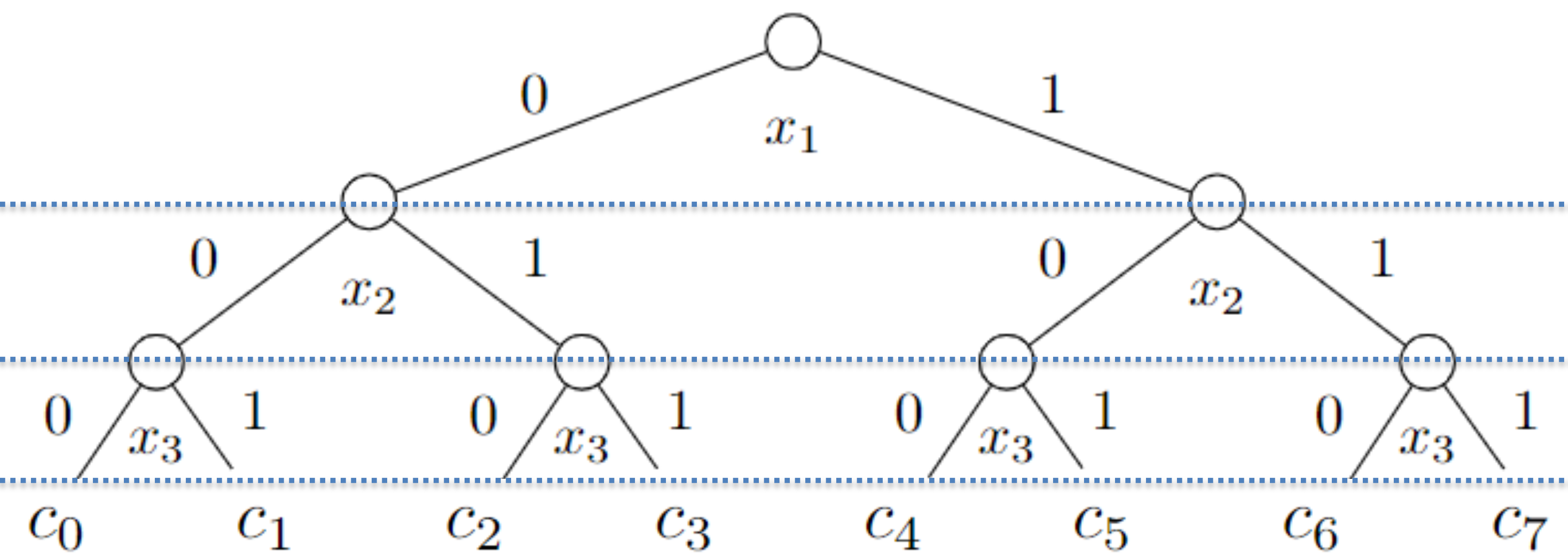


A 3-Qubit Quantum State

0 0 0	5%
0 0 1	2.5%
...	
1 1 0	5%
1 1 1	1.5%

Tree as a Quantum State

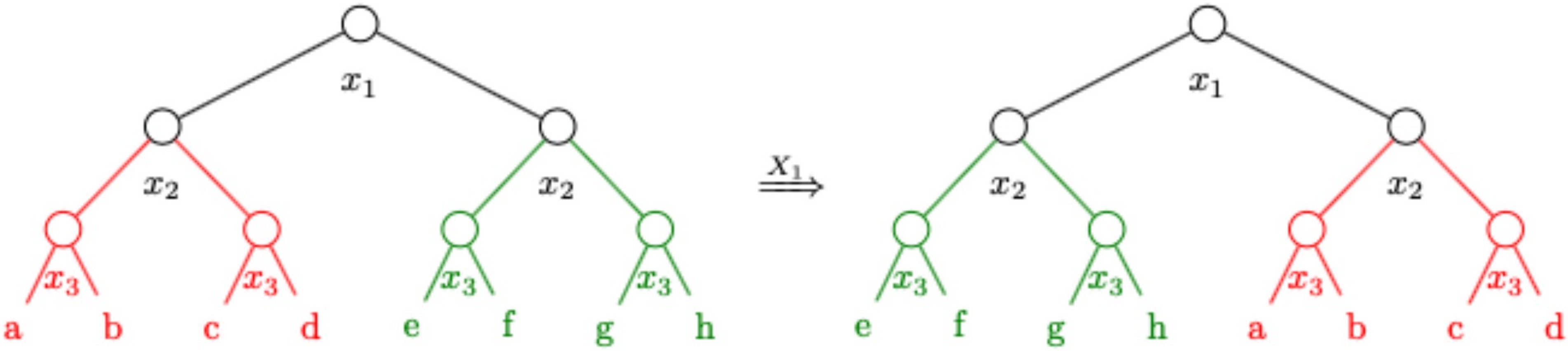
- A 3-bit quantum state



$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} \begin{matrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{matrix}$$

Quantum Gate and Tree Transformation

- An example of apply X gate (negation) on qubit x_1.



Quantum Circuit

- A sequence of quantum gates viewed as tree transformations

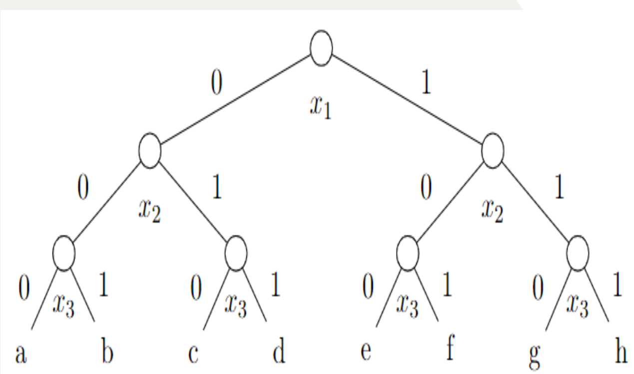
```
x q[0];
x q[1];
h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

```
cx q[1],q[2];
cx q[0],q[1];
cx q[2],q[3];
tdg q[0];
tdg q[1];
tdg q[2];
t q[3];
cx q[0],q[1];
cx q[2],q[3];
s q[3];
cx q[3],q[0];
h q[3];
```

```
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
cx q[1],q[2];
cx q[0],q[1];
cx q[2],q[3];
```

Why Quantum Circuit Verification?

- Increasing complexity of circuits
- Infeasibility of testing due to the probabilistic features




```
x q[0];
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h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

```
cx q[1],q[2];
cx q[0],q[1];
cx q[2],q[3];
tdg q[0];
tdg q[1];
tdg q[2];
t q[3];
cx q[0],q[1];
cx q[2],q[3];
s q[3];
cx q[3],q[0];
h q[3];
```

```
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
cx q[1],q[2];
cx q[0],q[1];
cx q[2],q[3];
```

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Outline

- Minimal Quantum Background and Motivation
- **Quantum Circuit Verification**
- Evaluation

Classical Hoare triple

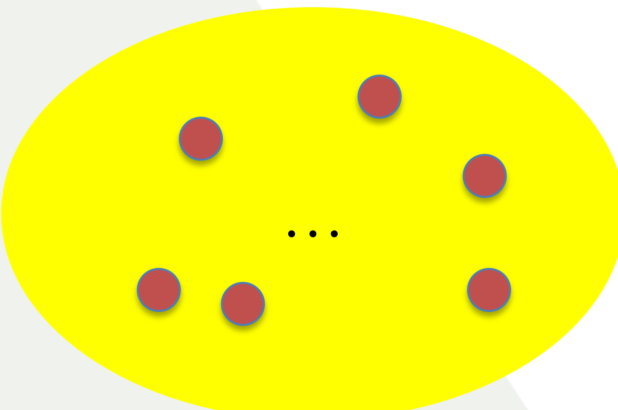
- For any predicates P and Q and any program S,

Precondition $\{P\}\ S\ \{Q\}$ Postcondition

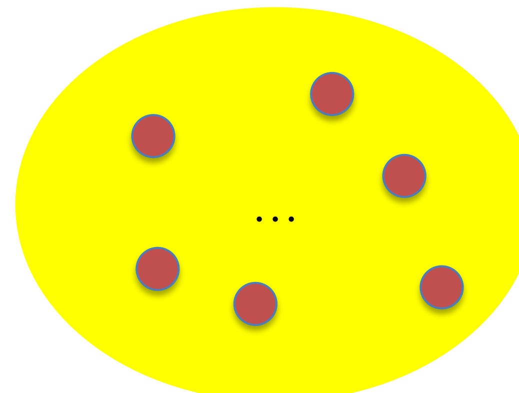
says that if S is started in (a state satisfying) P, then it terminates in Q.

Classical Hoare triple

- For any predicates P and Q and any program S,



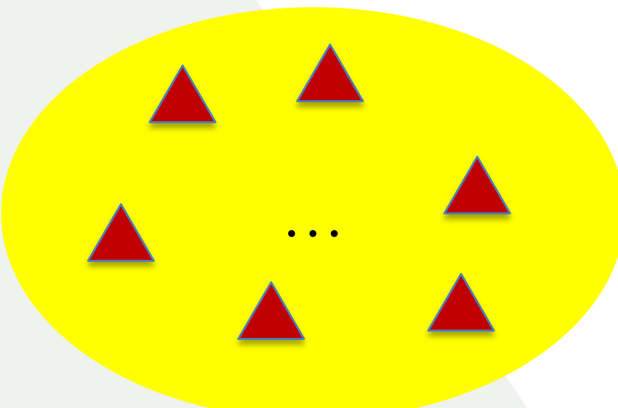
$$\{P\}\ S\ \{Q\}$$



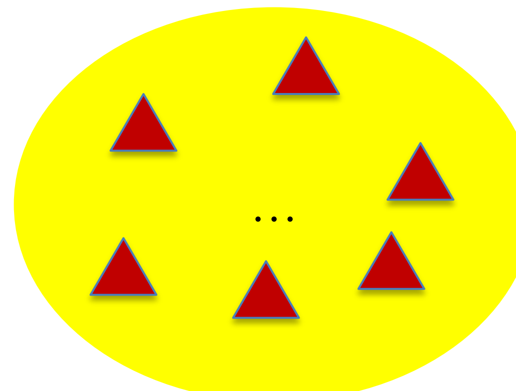
says that if S is started in (a state satisfying) P, then it terminates in Q.

Quantum Circuit Verification

- For any predicates P and Q and any circuit C,



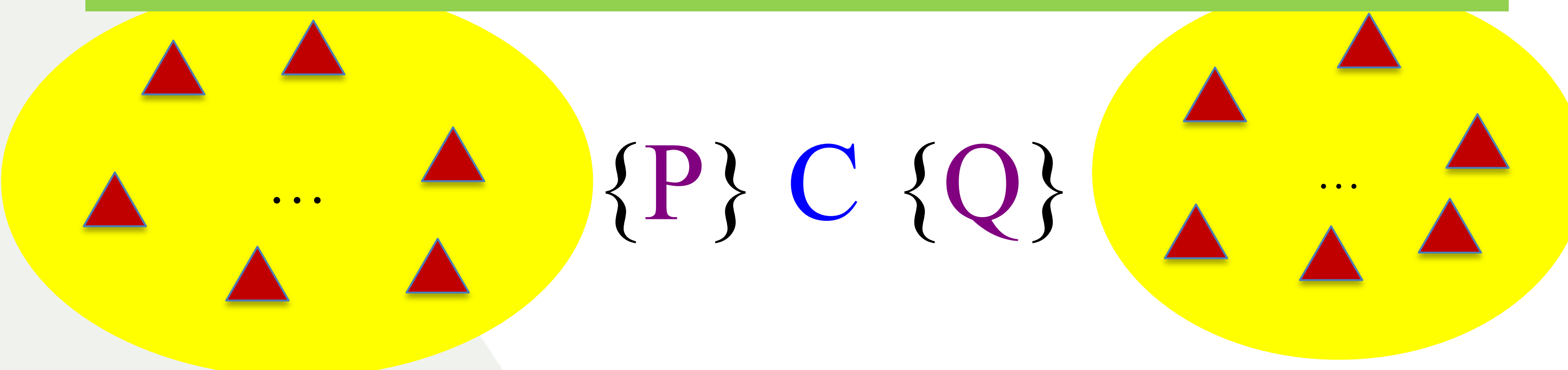
{P} C {Q}



says that if C is started in (a state satisfying) P, then it terminates in Q.

Quantum Circuit Verification

Need a symbolic representation of a set of quantum states (trees).



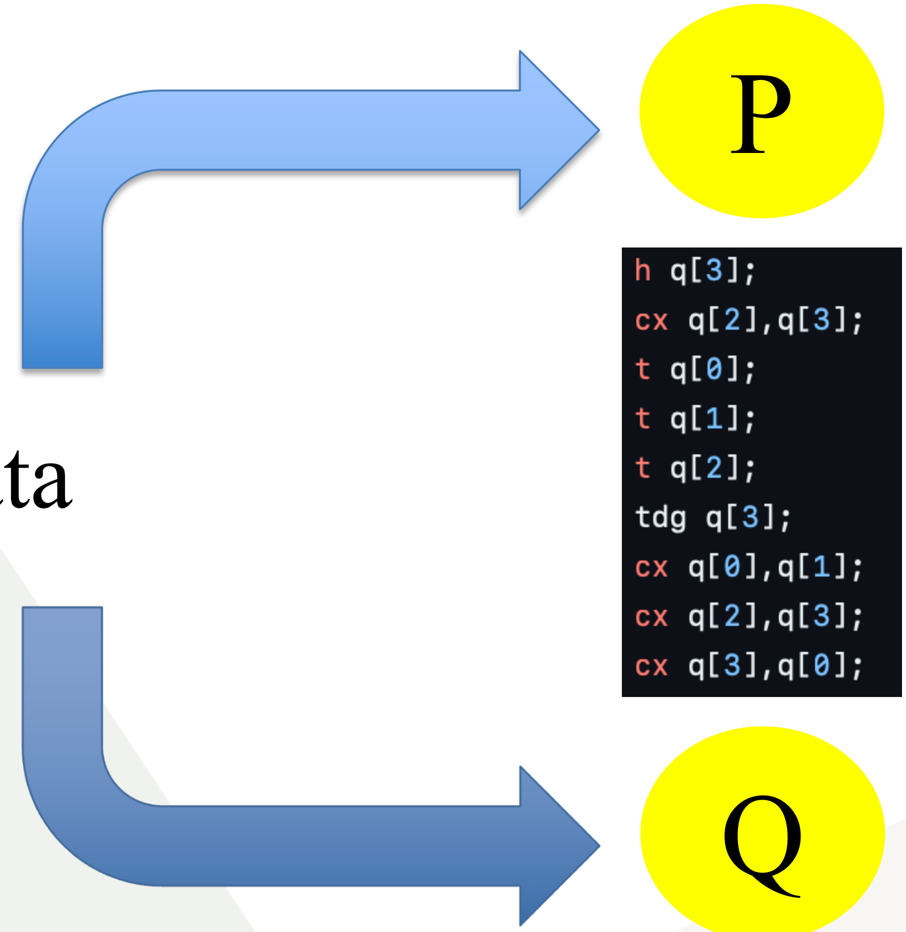
From automata theory:

Set of words ➔ Regular language (Finite automata)

Set of trees ➔ Regular tree language (Tree automata)

Overview

Tree automata




```
h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

Overview

P^1

P

```
h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

Q

Overview

P^2

P

```
h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

Q

Overview

P

P^3

```
h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

Q

Overview

P

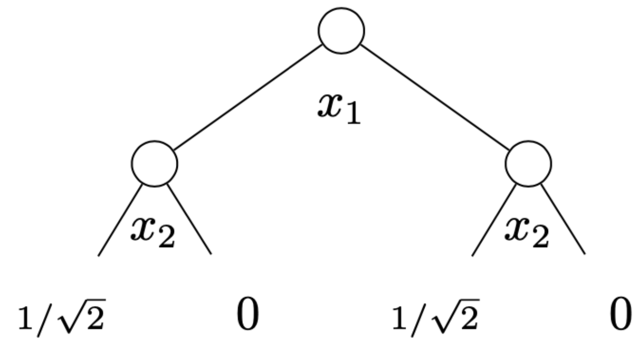
```
h q[3];
cx q[2],q[3];
t q[0];
t q[1];
t q[2];
tdg q[3];
cx q[0],q[1];
cx q[2],q[3];
cx q[3],q[0];
```

$P^{10} \subseteq Q$

(via standard TA algorithms)

Examples: Tree Automata of Only One Quantum State

- This TA of the quantum state



Probability $= |1/\sqrt{2}|^2 = 1/2$

$$q \xrightarrow{\boxed{x_1}} (q_0, q_0) \qquad q_1 \xrightarrow{\boxed{1/\sqrt{2}}} ()$$

$$q_0 \xrightarrow{\boxed{x_2}} (q_1, q_2) \qquad q_2 \xrightarrow{\boxed{0}} ()$$

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Examples: Tree Automata Encoding of Quantum States

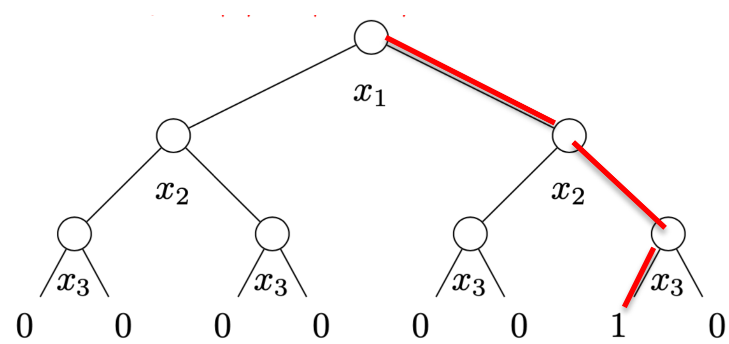
- This TA accepts all 3-qubit basis quantum states

$q \xrightarrow{x_1} (r_0, r_1)$ $\quad r_1 \xrightarrow{x_2} (s_0, s_1)$ $\quad s_1 \xrightarrow{x_3} (q_0, q_1)$ $\quad q_0 \xrightarrow{\mathbf{0}} ()$

$q \xrightarrow{x_1} (r_1, r_0)$ $\quad r_1 \xrightarrow{x_2} (s_1, s_0)$ $\quad s_1 \xrightarrow{x_3} (q_1, q_0)$ $\quad q_1 \xrightarrow{\mathbf{1}} ()$

$r_0 \xrightarrow{x_2} (s_0, s_0)$ $\quad s_0 \xrightarrow{x_3} (q_0, q_0)$

- $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle,$
  $|100\rangle, |101\rangle, {\color{red}|110\rangle}, |111\rangle\}$



TA as Compact Representation of Quantum States

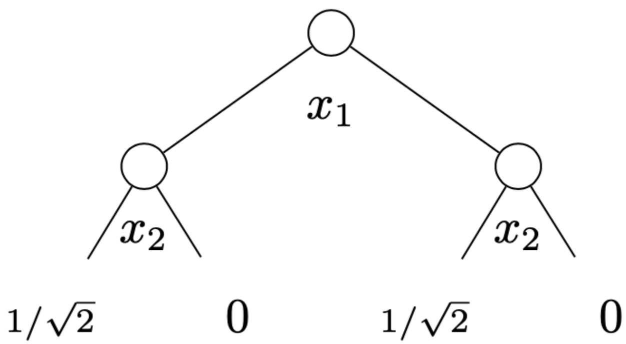
- This TA accepts all 2^n basis states.
  # of transitions: $3n+1$

$q \xrightarrow{x_1} (r_0, r_1)$	$r_1 \xrightarrow{x_2} (s_0, s_1)$	$\dots$	$z_1 \xrightarrow{x_n} (q_0, q_1)$	$q_0 \xrightarrow{\mathbf{0}} ()$
$q \xrightarrow{x_1} (r_1, r_0)$	$r_1 \xrightarrow{x_2} (s_1, s_0)$	$\dots$	$z_1 \xrightarrow{x_n} (q_1, q_0)$	$q_1 \xrightarrow{\mathbf{1}} ()$
	$r_0 \xrightarrow{x_2} (s_0, s_0)$	$\dots$	$z_0 \xrightarrow{x_n} (q_0, q_0)$	

Why it can be some compact? Merge shared structures.

Examples of Gate Operations:

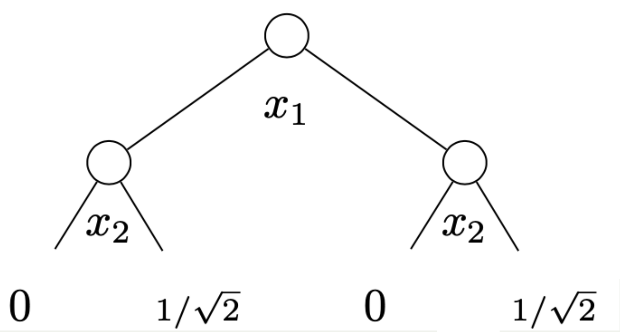
X gate on qubit 2.



$$q - \boxed{x_1} \rightarrow (q_0, q_0) \qquad q_1 - \boxed{1/\sqrt{2}} \rightarrow ()$$

$$q_0 - \boxed{x_2} \rightarrow (q_1, q_2) \qquad q_2 - \boxed{0} \rightarrow ()$$

$$\overset{X_2}{\Rrightarrow}$$

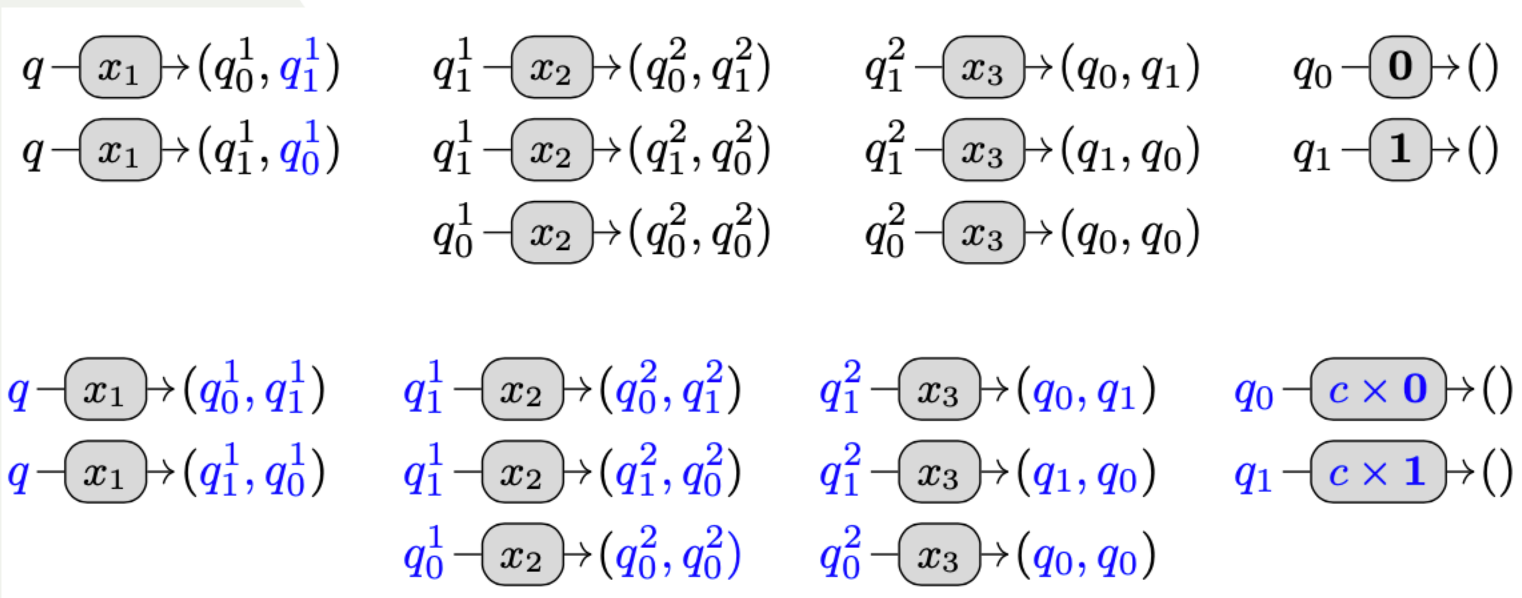


$$q - \boxed{x_1} \rightarrow (q_0, q_0) \qquad q_1 - \boxed{1/\sqrt{2}} \rightarrow ()$$

$$q_0 - \boxed{x_2} \rightarrow ({\color{blue}q_2, q_1}) \qquad q_2 - \boxed{0} \rightarrow ()$$

Example of Gate Operations: Z, S, T gates on qubit 1.

- Multiply the right subtree of x_1 with some constant c.



Example of Gate Operations:

Z, S, T gates on qubit 1.

- Multiply the right subtree of x_1 with some constant c

Other gates? In the paper.

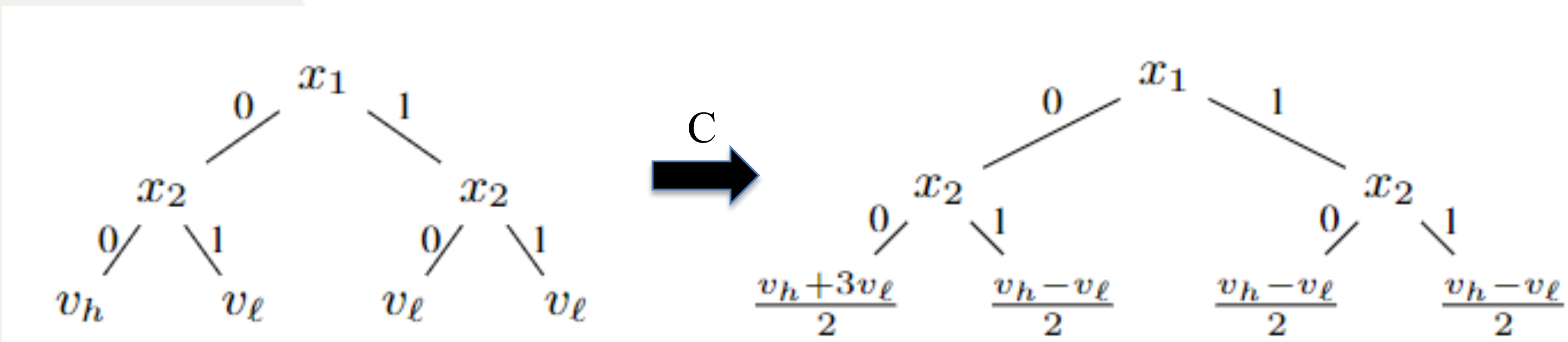
Our framework allows all quantum gates supported in conventional quantum computers.

$$q \rightarrow x_1(q_0^1, q_1^1) \quad q_1^1 \rightarrow x_2(q_0^2, q_1^2) \quad q_1^2 \rightarrow x_3(q_0, q_1) \quad q_0 \rightarrow c \times \mathbf{0}()$$

$$q \rightarrow x_1(q_1^1, q_0^1) \quad q_1^1 \rightarrow x_2(q_1^2, q_0^2) \quad q_1^2 \rightarrow x_3(q_1, q_0) \quad q_1 \rightarrow c \times \mathbf{1}()$$

$$q_0^1 \rightarrow x_2(q_0^2, q_0^2) \quad q_0^2 \rightarrow x_3(q_0, q_0)$$

Extension: Symbolic Tree



A Symbolic Tree (State), s

The state after executing C, C(s)

Yu-Fang Chen, Kai-Min Chung, Ondřej Lengál, Jyun-Ao Lin, Wei-Lun Tsai.
AutoQ: An Automata-based Quantum Circuit Verifier.
CAV 2023

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Outline

- Minimal Quantum Background and Motivation
- Quantum Circuit Verification in the Hoare-Style
- **Evaluation**

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Experiment - Verification

- BV, Grover-Sing: $|P| = 1$
- MCToffoli, Grover-All: $|P| \gg 1$

				AutoQ-Hybrid	
	n	#q	#G	analysis	=
BV	95	96	241	6.0s	0.0s
	96	97	243	5.9s	0.0s
	97	98	246	6.3s	0.0s
	98	99	248	6.5s	0.0s
	99	100	251	6.7s	0.0s
Grover-Sing	12	24	5,215	11s	0.0s
	14	28	12,217	31s	0.0s
	16	32	28,159	1m29s	0.0s
	18	36	63,537	4m1s	0.0s
	20	40	141,527	10m56s	0.0s
MCToffoli	8	16	15	0.0s	0.0s
	10	20	19	0.0s	0.0s
	12	24	23	0.0s	0.0s
	14	28	27	0.1s	0.0s
	16	32	31	0.2s	0.0s
Grover-All	6	18	357	3.3s	0.0s
	7	21	552	10s	0.0s
	8	24	939	39s	0.1s
	9	27	1,492	2m17s	0.4s
	10	30	2,433	9m48s	2.1s

Experiment – Bug Hunting

- Create C_{bug} by appending one random gate at a random qubit to C_{ori}.
- Check if $C_{bug} \neq C_{ori}$ by testing them with different initial state sets P.
- Useful in compiler validation

Experiment – Bug Hunting

	circuit	#q	#G	AutoQ time	AutoQ bug?	Feynman time	Feynman bug?	Qcec time	Qcec bug?
FeynmanBench	csum_mux_9	30	141	0.5s	T	6.0s	–	1m1s	F
	gf2^10_mult	30	348	1.5s	T	0.5s	–	1.6s	–
	gf2^16_mult	48	876	9.3s	T	3.6s	–	1m25s	T
	gf2^32_mult	96	3,323	2m0s	T	51s	–	2m52s	T
	ham15-high	20	1,799	7.5s	T	3m50s	–	56.8s	T
	mod_adder_1024	28	1,436	10s	T	8.7s	–	1m2s	T
Random	35a	35	106	2.6s	T	0.2s	–	1m4s	F
	35b	35	106	1.3s	T	0.1s	T	1m5s	F
	35c	35	106	1.1s	T	0.1s	T	1m7s	T
	35d	35	106	1.1s	T	0.1s	T	1m5s	T
	35e	35	106	1.0s	T	0.1s	–	1m4s	T
	35f	35	106	2.0s	T	0.2s	T	1m5s	F
	35g	35	106	1.0s	T	0.2s	–	1m7s	T
	35h	35	106	1.2s	T	0.2s	–	2.0s	–
	35i	35	106	1.2s	T	0.3s	T	1m6s	T
	35j	35	106	1.4s	T	0.2s	–	1m6s	F
RevLib	add16_174	49	65	3.1s	T	timeout		1m16s	T
	add32_183	97	129	25s	T	timeout		2m2s	T
	add64_184	193	257	2m8s	T	timeout		2.0s	–
	avg8_325	320	1,758	22m42s	T	timeout		2.2s	–
	bw_291	87	308	11s	T	15s	T	2m10s	T
	cycle10_293	39	79	0.5s	T	0.5s	T	1m10s	T
	e64-bdd_295	195	388	50s	T	timeout		1.5s	–
	ex5p_296	206	648	2m5s	T	1m36s	T	2.0s	–
	ham15_298	45	154	1.3s	T	0.9s	T	1m15s	T
	mod5adder_306	32	97	0.8s	T	1.1s	T	1m2s	T
	rd84_313	34	105	0.9s	T	1.5s	T	1m5s	T

circuit	#q	#G	AutoQ time	AutoQ bug?	Feynman time	Feynman bug?	Qcec time	Qcec bug?
hwb10	16	31,765	1m49s	T	timeout		50.6s	T
hwb11	15	87,790	4m31s	T	timeout		48.8s	T
hwb12	20	171,483	13m43s	T	timeout		1m30s	T
hwb8	12	6,447	16s	T	timeout		43.6s	T
qcla_adder_10	36	182	1.7s	T	1.0s	–	1m5s	F
qcla_mod_7	26	295	2.0s	T	1m28s	–	59.0s	F
70a	70	211	21s	T	1.3s	–	1m 42s	T
70b	70	211	15s	T	0.7s	T	1m41s	T
70c	70	211	10s	T	0.8s	–	1m37s	T
70d	70	211	timeout		0.9s	T	1m39s	T
70e	70	211	24s	T	0.8s	–	1m36s	T
70f	70	211	31s	T	0.8s	T	1m34s	F
70g	70	211	16m8s	T	1.2s	–	1m47s	T
70h	70	211	15s	T	1.3s	–	1m42s	T
70i	70	211	18s	T	1.0s	–	1m47s	T
70j	70	211	1m37s	T	1.2s	–	1m49s	T
urf1_149	9	11,555	40s	T	timeout		45.2s	T
urf2_152	8	5,031	14s	T	21m39s	T	36.4s	T
urf3_155	10	26,469	1m29s	T	timeout		45.1s	T
urf4_187	11	32,005	2m10s	T	timeout		46.6s	T
urf5_158	9	10,277	26s	T	timeout		38.2s	T
urf6_160	15	10,741	1m6s	T	timeout		48.4s	T
hwb6_301	46	160	2.5s	T	2.7s	T	1m13s	T
hwb7_302	73	282	10s	T	15s	T	1m39s	T
hwb8_303	112	450	34s	T	43s	T	2m25s	T
hwb9_304	170	700	1m48s	T	2m29s	T	2.0s	–

https://github.com/meamy/feynman
https://github.com/cda-tum/qcec

Thank You

Summary:

- Interesting link between automata and quantum states.
- So far only basic TA has been tried, there are many more possibilities.
- The main reason for efficiency, the compact structure of TA.
- Welcome for more discussions!

State-of-the-Art

We focus on **fully** automatic approaches:

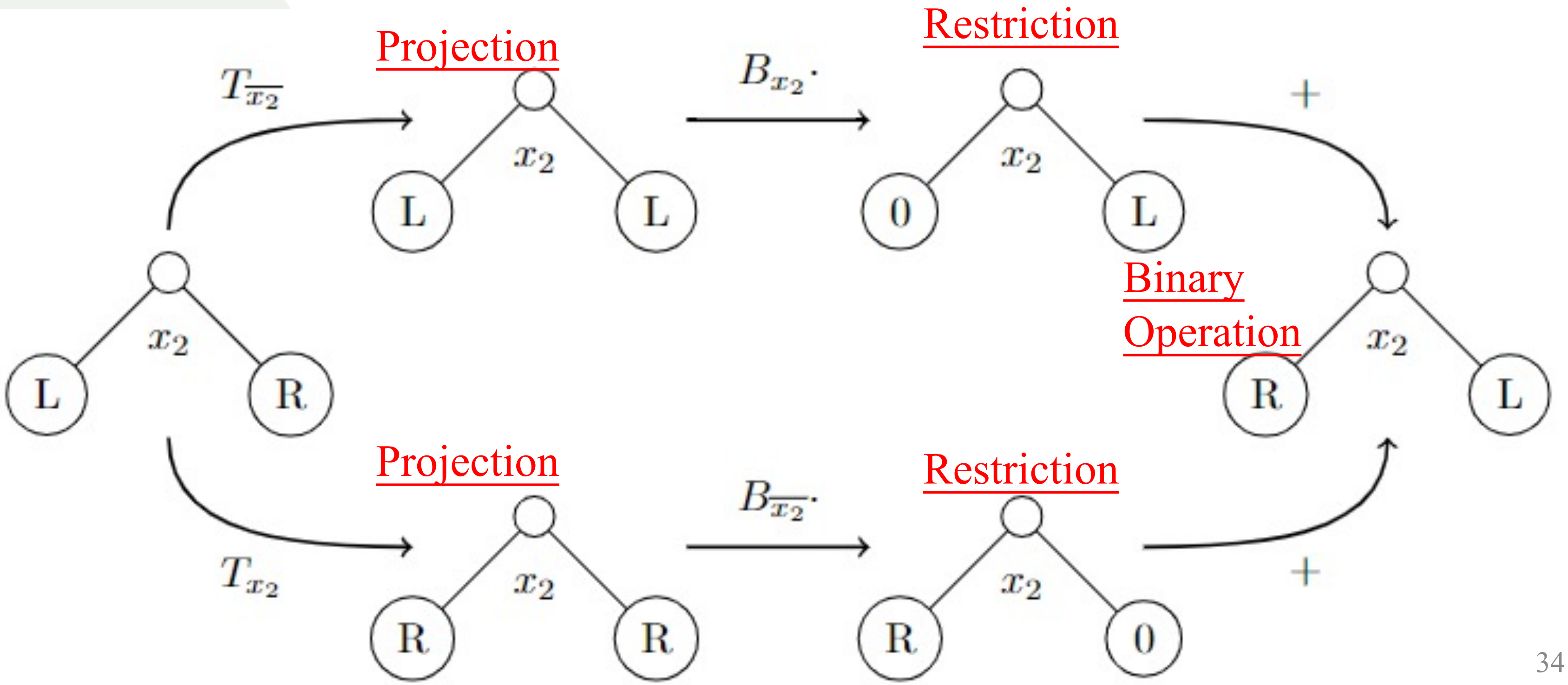
- **Quantum Simulation**
  - low coverage
- **Quantum Abstraction Interpretation**
  - over approximation and cannot catch bugs
- **Quantum Model Checking**
  - only work for small examples
- **Circuit Equivalence Checking**
  - inflexible for user custom properties

Two Approaches for TA Gate Operations

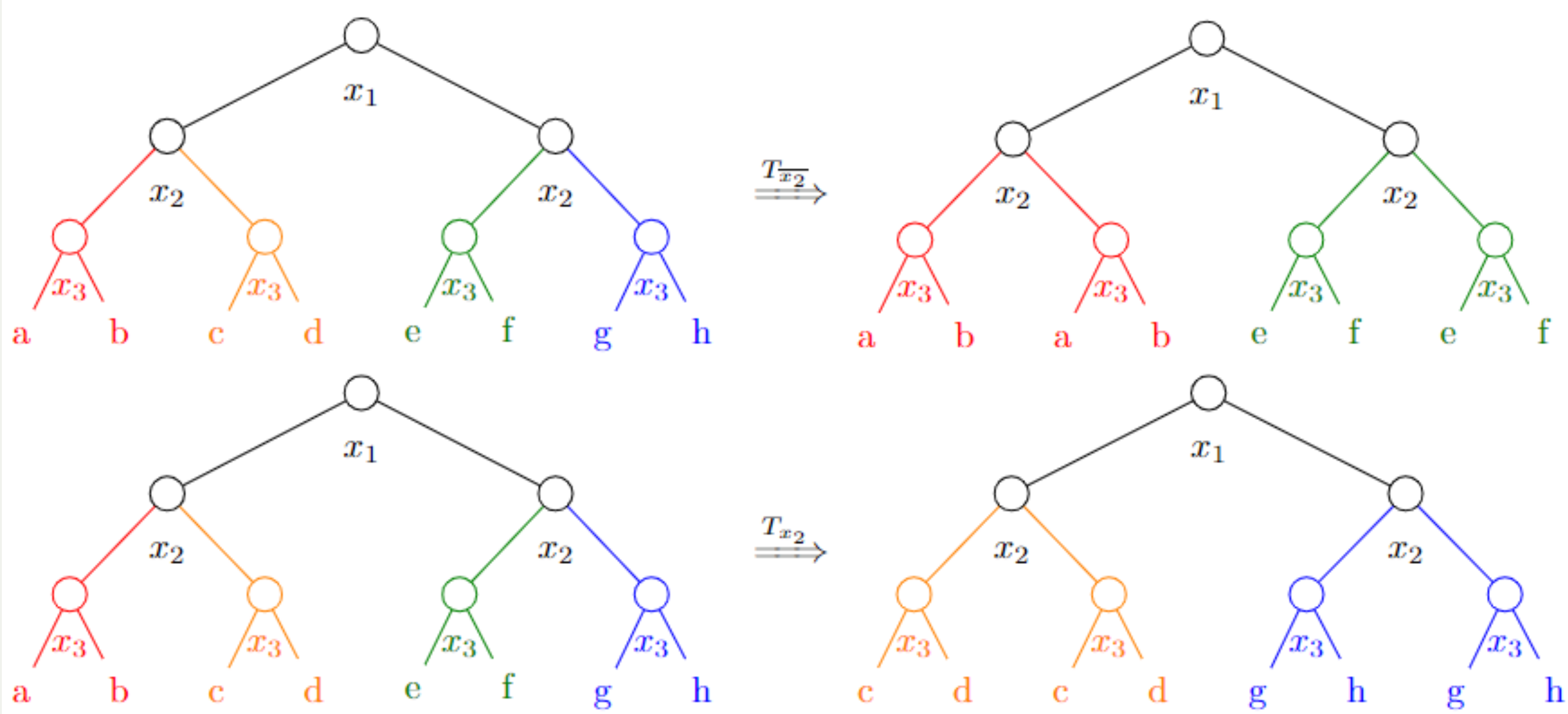
- Permutation-based approach:
  - Faster, but works for a smaller set of gates.
  - Done by directly modifying TA transitions.
- Composition-based approach
  - Slower, but complete for universal quantum computing.

X gate operating on qubit 2

- $B_{x_2} \cdot T_{\overline{x_2}} + B_{\overline{x_2}} \cdot T_{x_2}$

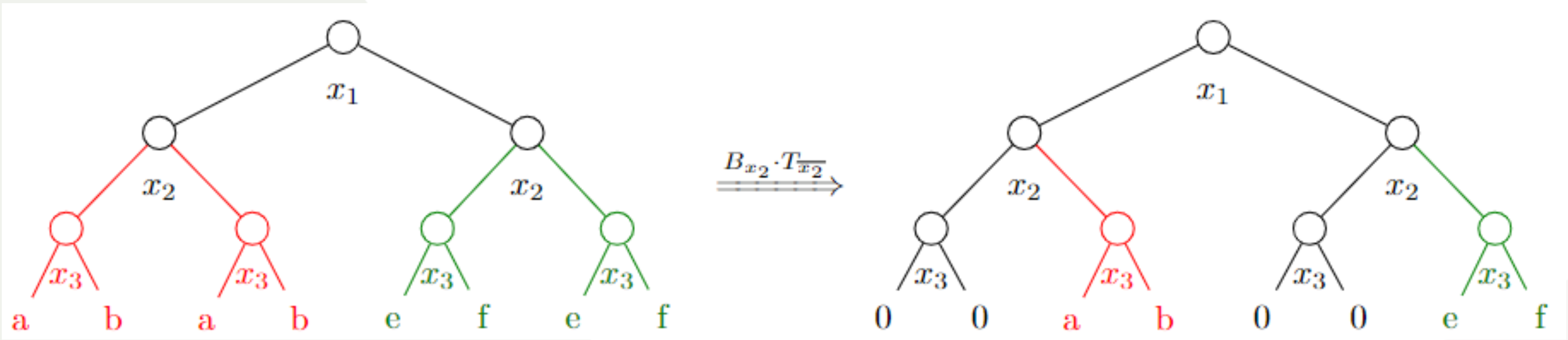


1. Projection ($T_{\overline{x_2}}$ & T_{x_2})



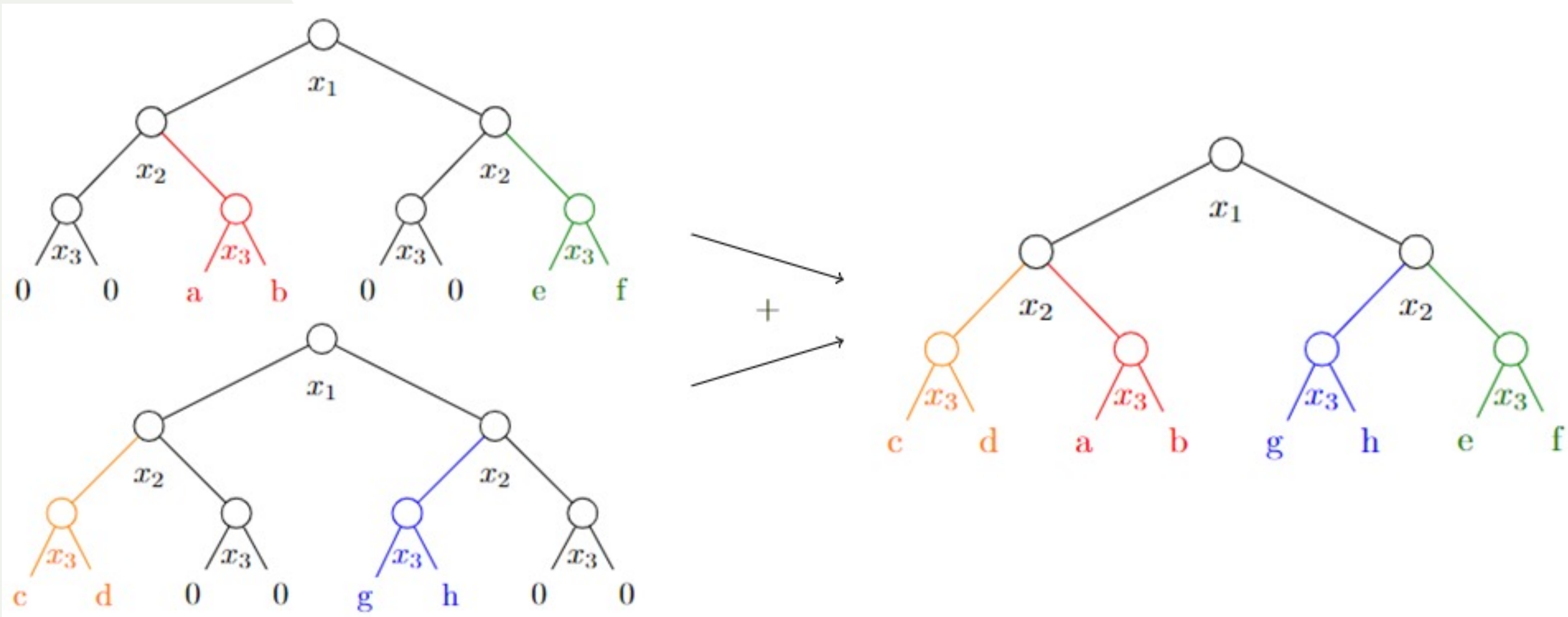
2. Restriction ($B_{x_2}\cdot$ & $B_{\overline{x_2}}\cdot$)

- $B_{x_2} \cdot T_{\overline{x_2}}$



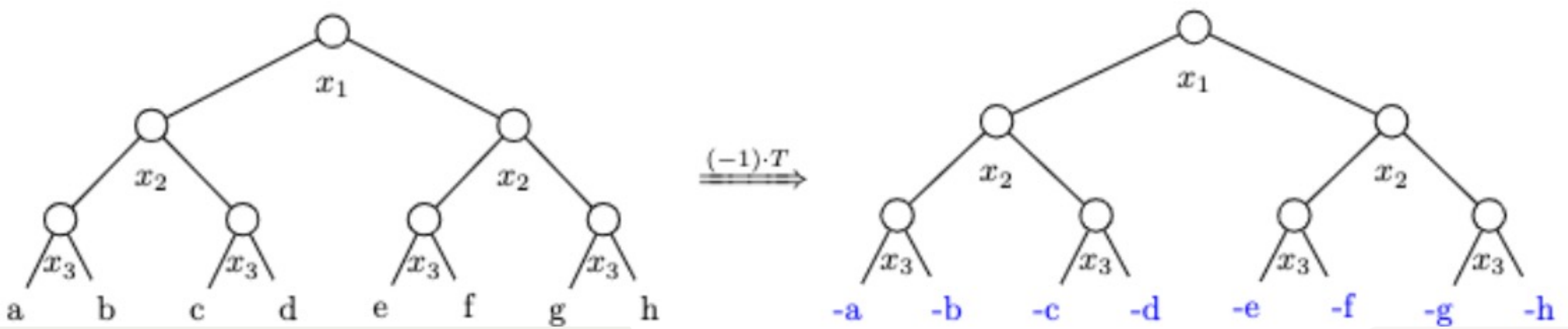
3. Binary Operation (+ & –)

- $B_{x_2} \cdot T_{\overline{x_2}} + B_{\overline{x_2}} \cdot T_{x_2}$



4. Constant Multiplication (c ·)

- (-1)· *T*



Tree Operations

Table 1. Symbolic update formulae for the considered quantum gates. Notice that in all cases x_c and x'_c denote the two control bits, and x_t and x'_t denote the two target bits, if they exist.

Gate	Update
X_t	$B_{x_t} \cdot T_{\overline{x_t}} + B_{\overline{x_t}} \cdot T_{x_t}$
Y_t	$\omega^2 \cdot (B_{x_t} \cdot T_{\overline{x_t}} - B_{\overline{x_t}} \cdot T_{x_t})$
Z_t	$B_{\overline{x_t}} \cdot T - B_{x_t} \cdot T$
H_t	$(T_{\overline{x_t}} + B_{\overline{x_t}} \cdot T_{x_t} - B_{x_t} \cdot T)/\sqrt{2}$
S_t	$B_{\overline{x_t}} \cdot T + \omega^2 \cdot B_{x_t} \cdot T$
T_t	$B_{\overline{x_t}} \cdot T + \omega \cdot B_{x_t} \cdot T$
$\mathrm{Rx}(\frac{\pi}{2})_t$	$(T - \omega^2 \cdot (B_{x_t} \cdot T_{\overline{x_t}} + B_{\overline{x_t}} \cdot T_{x_t}))/\sqrt{2}$
$\mathrm{Ry}(\frac{\pi}{2})_t$	$(T_{\overline{x_t}} + B_{x_t} \cdot T - B_{\overline{x_t}} \cdot T_{x_t})/\sqrt{2}$
CNOT_t^c	$B_{\overline{x_c}} \cdot T + B_{x_c} \cdot B_{\overline{x_t}} \cdot T_{x_t} + B_{x_c} \cdot B_{x_t} \cdot T_{\overline{x_t}}$
CZ_t^c	$B_{\overline{x_c}} \cdot T + B_{\overline{x_t}} \cdot T - B_{\overline{x_c}} \cdot B_{\overline{x_t}} \cdot T - B_{x_c} \cdot B_{x_t} \cdot T$
$\mathrm{Toffoli}_t^{c,c'}$	$B_{\overline{x_c}} \cdot T + B_{\overline{x_{c'}}} \cdot T - B_{\overline{x_c}} \cdot B_{\overline{x_{c'}}} \cdot T + B_{x_t} \cdot B_{x_c} \cdot B_{x_{c'}} \cdot T_{\overline{x_t}} + B_{\overline{x_t}} \cdot B_{x_c} \cdot B_{x_{c'}} \cdot T_{x_t}$
$\mathrm{Fredkin}_{t,t'}^c$	$B_{\overline{x_c}} \cdot T + B_{x_t} \cdot B_{x_{t'}} \cdot B_{x_c} \cdot T + B_{\overline{x_t}} \cdot B_{\overline{x_{t'}}} \cdot B_{x_c} \cdot T + B_{x_t} \cdot B_{\overline{x_{t'}}} \cdot B_{x_c} \cdot T_{\overline{x_t} x_{t'}} + B_{\overline{x_t}} \cdot B_{x_{t'}} \cdot B_{x_c} \cdot T_{x_t \overline{x_{t'}}}$

Yuan-Hung Tsai, Jie-Hong R. Jiang, and Chiao-Shan Jhang. 2021. Bit-Slicing the Hilbert Space: Scaling Up Accurate Quantum Circuit Simulation. In *58th ACM/IEEE Design Automation Conference, DAC 2021, San Francisco, CA, USA, December 5-9, 2021*. IEEE, 439–444. https://doi.org/10.1109/DAC18074.2021.9586191

Lift these operations to TA

- Constant Multiplication
- Restriction
- Binary Operation
- Projection
  - need to say the same left and right subtrees.