An Executable Sequential Specification for SPARK Aggregation

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Current Trends in CAV (Computer-Aided Verification)

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- verification of HW

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  - ...
- verification of concurrent programs
  - mutual exclusion protocols
  - concurrent data structures
  - ...

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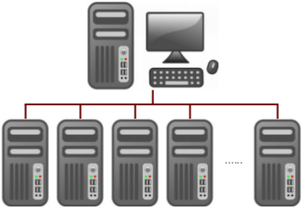
    - expected over $50 billion by 2020



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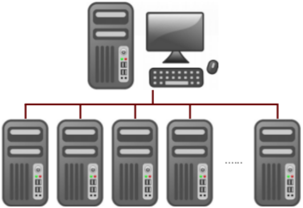

- Can verification be applied here, too?

Frameworks for Big Data

Programming for Big Data:

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- examples:
  - Hadoop MAPREDUCE
  - PIG
  - HIVE
  - **Apache SPARK**

Frameworks for Big Data

New verification problems:

Frameworks for Big Data

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- verification of correctness of the frameworks

Frameworks for Big Data

New verification problems:

- verification of correctness of the frameworks
- verification of correctness of user programs
  - correctness: checking special properties

Apache SPARK

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  - claims to be up to 100$\times$ faster due to in-memory computation
- a relaxed fault tolerant model
  - sub-results are recomputed upon faults
- lazy evaluation semantics
- contains libraries for
  - processing graphs
  - streaming computation
  - machine learning
  - SQL-based database computation
  - ...

Apache SPARK

RDD—Resilient Distributed Dataset:

- the principal data abstraction

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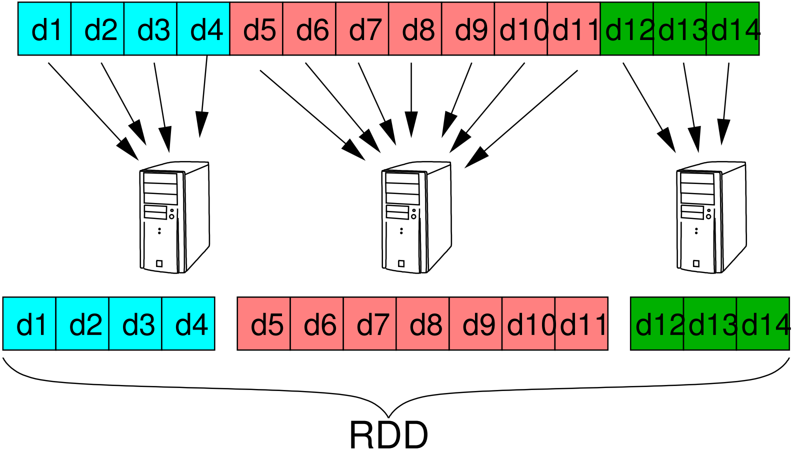

Computation in SPARK

- map-style
  - **map**, **filter**

```
RDD rdd = ...
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- combined
  - **aggregateByKey**, **reduceByKey**

```
PairRDD rdd = ...
PairRDD sum = rdd.reduceByKey(λ x y . x + y)
```

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  - multi-paradigm programming language
  - both imperative and functional code (side-effects)
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  - correctness requirements on aggregation functions (next slide)
  - analysis of case studies—finding numeric instability in ML library

Commutativity in SPARK aggregation

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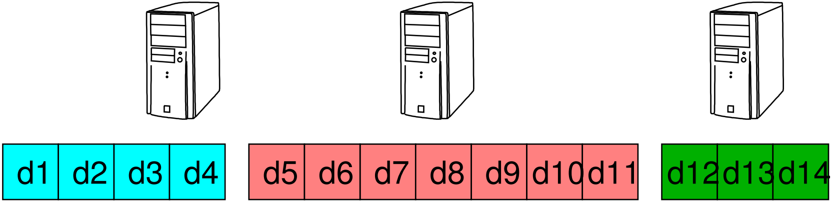
$$\texttt{aggregate}(seq, comb, \perp, rdd)$$



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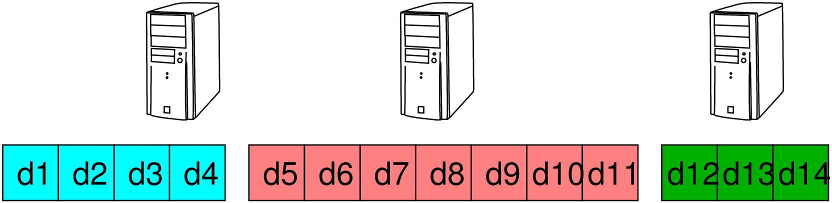
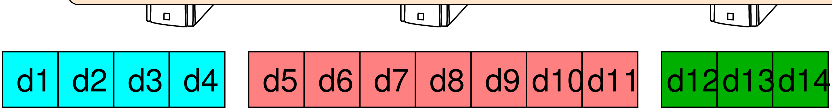


$$\texttt{foldl}(seq, \bot, [d_1, d_2, d_3, d_4]) \quad \rightsquigarrow r_A$$
$$\texttt{foldl}(seq, \bot, [d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}]) \quad \rightsquigarrow r_B$$
$$\texttt{foldl}(seq, \bot, [d_{12}, d_{13}, d_{14}]) \quad \rightsquigarrow r_C$$

Commu

$$\texttt{foldl} :: (B \to A \to B) \to B \to [A] \to B$$
$$\texttt{foldl}(seq, \bot, [\,]) = \bot$$
$$\texttt{foldl}(seq, \bot, x : xs) = \texttt{foldl}(seq, \underline{seq(\bot, x)}, xs)$$



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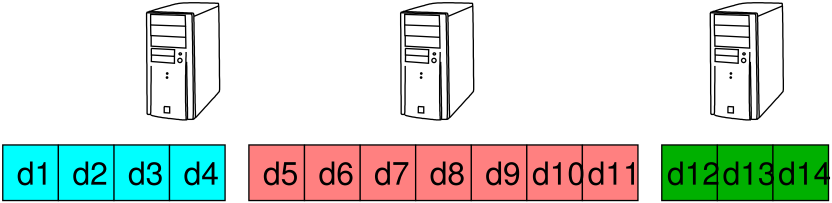

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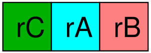
- collected results: 

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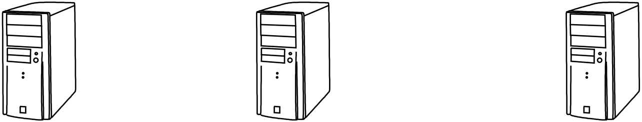

d1	d2	d3	d4		d5	d6	d7	d8	d9	d10	d11		d12	d13	d14

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- collected results: | rC | rA | rB |

nondeterministic!

Commutativity in SPARK aggregation

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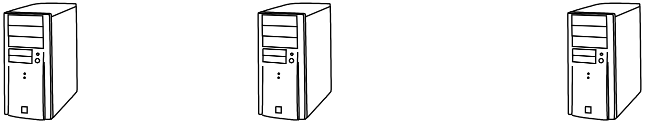

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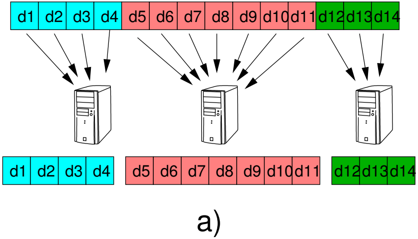
1. Partitioning into RDD



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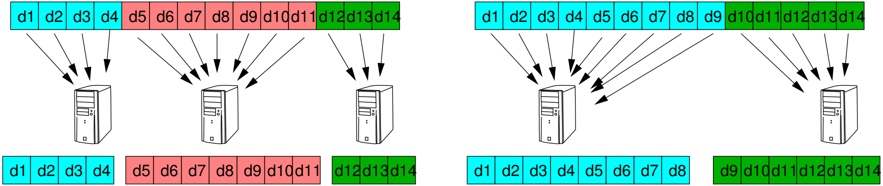

a) b)

2. Order in which nodes send partial results
   - rC rA rB
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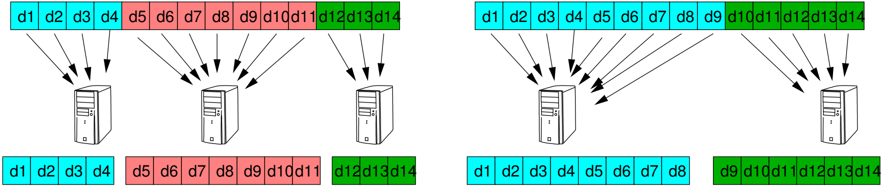

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`aggregate` can yield different results!!!

Commutativity in SPARK aggregation

Example of a nondeterministic aggregation

$$\texttt{aggregate}(\mathit{seq}, \mathit{comb}, \perp, \mathit{rdd})$$

$$\begin{aligned} \mathit{seq}(\mathit{acc}, x) &= \mathit{acc} + x \\ \mathit{comb}(\mathit{lhs}, \mathit{rhs}) &= \mathit{rhs} + \mathit{rhs} \qquad (\mathit{typo}) \\ \perp &= 0 \end{aligned}$$

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$\texttt{foldl}(seq, 0, [d_1, d_2, d_3, d_4]) = (((0 + d_1) + d_2) + d_3) + d_4 \quad \rightsquigarrow r_A$

$\texttt{foldl}(seq, 0, [d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}]) = \ldots \quad \rightsquigarrow r_B$

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Collecting results:

- rC rA rB : $\texttt{\textbf{foldl}}(comb, 0, [r_C, r_A, r_B]) = 2r_B$
- rA rB rC : $\texttt{\textbf{foldl}}(comb, 0, [r_A, r_B, r_C]) = 2r_C$

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$$\texttt{aggregate}(seq, comb, \bot, rdd)$$

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$$2r_B \neq 2r_C$$

Commutativity in SPARK aggregation

Commutativity of `aggregate`

Definition

A call

$$\texttt{aggregate}(\mathit{seq}, \mathit{comb}, \perp, \mathit{rdd})$$

is **commutative** iff

$$\texttt{aggregate}(\mathit{seq}, \mathit{comb}, \perp, \mathit{rdd}(L)) = \texttt{foldl}(\mathit{seq}, \perp, L)$$

for every partitioning $\mathit{rdd}(L)$ of L.

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- i.e., **`aggregate`** is an implementation of **`foldl`**
- if a call to **`aggregate`** is commutative:
  - the call is deterministic
  - when analyzing the program, we can assume one partitioning

Commutativity in SPARK aggregation

Conditions for commutative `aggregate`

Theorem

Consider rdd of elements of type $\mathbb{T}$ and $\bot \in \mathbb{R}$. A call

$$\texttt{aggregate}(seq, comb, \bot, rdd)$$

is commutative iff

1. $(img(\texttt{foldl}(seq, \bot)), comb, \bot)$ *is a commutative monoid and*
2. *for all $d \in \mathbb{T}$ and $e \in img(\texttt{foldl}(seq, \bot))$, it holds that*
$$seq(e, d) = comb(e, seq(z, d)).$$

Commutativity in SPARK aggregation

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Safe approximation:

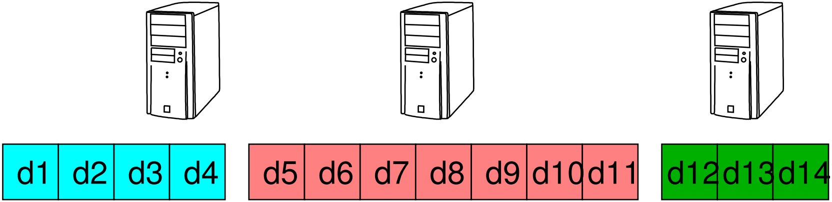
…is commutative if

1. $(\mathbb{R}, comb, \perp)$ is a commutative monoid and
2. for all $d \in \mathbb{T}$ and $e \in \mathbb{R}$, it holds that
$$seq(e, d) = comb(e, seq(z, d)).$$

Commutativity in SPARK aggregation

SPARK `reduce`

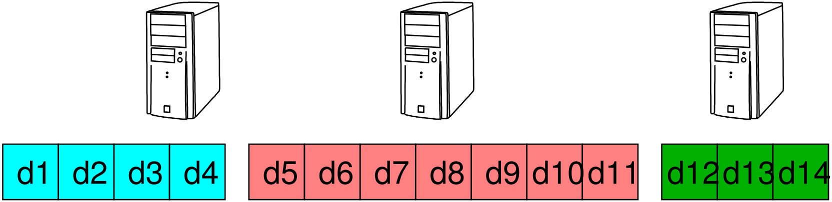
$$\texttt{reduce}(\mathit{comb}, \mathit{rdd})$$



Commutativity in SPARK aggregation

SPARK reduce

reduce(*comb*, *rdd*)



$\texttt{reducel}(comb, [d_1, d_2, d_3, d_4]) \rightsquigarrow r_A$

$\texttt{reducel}(comb, [d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}]) \rightsquigarrow r_B$

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Commutativity in SPARK aggregation

SPARK r

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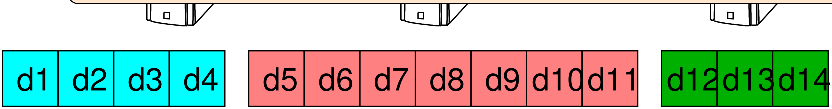
$$\texttt{reducel}(comb, x : xs) = \texttt{foldl}(comb, x, xs)$$



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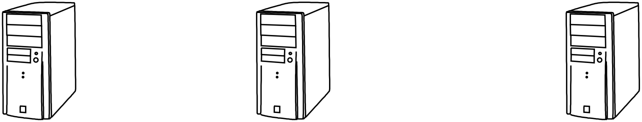

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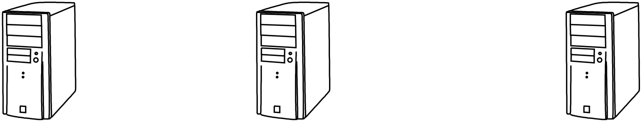


d1	d2	d3	d4

d5	d6	d7	d8	d9	d10	d11

d12	d13	d14

reduceI$(comb, [d_1, d_2, d_3, d_4]) \rightsquigarrow r_A$

reduceI$(comb, [d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}]) \rightsquigarrow r_B$

reduceI$(comb, [d_{12}, d_{13}, d_{14}]) \rightsquigarrow r_C$

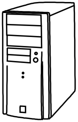
- collected results: | rC | rA | rB |

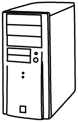
nondeterministic!

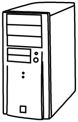
Commutativity in SPARK aggregation

SPARK `reduce`

reduce(*comb*, *rdd*)







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$\texttt{reducel}(comb, [r_C, r_A, r_B]) \rightsquigarrow$ *result*

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Conditions for commutative `reduce`

$$\texttt{reduce}(\mathit{comb}, \mathit{rdd})$$

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$seq_2(x, y) = \textbf{case } x \textbf{ of}$
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  $\textbf{Just } v \rightarrow \textbf{Just } comb(v, y)$

$comb_2(x, y) = \textbf{case } (x, y) \textbf{ of}$
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Theorem

Consider rdd of elements of type $\mathbb{T}$. A call

$$\texttt{reduce}(comb, rdd)$$

is commutative iff $(\mathbb{T}, comb)$ is a commutative semigroup.

Commutativity in SPARK aggregation

SPARK `treeAggregate` and `treeReduce`

- first stage same as **`aggregate`** and **`reduce`**

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Theorem

- ***`treeAggregate`***$(seq, comb, \perp, rdd)$ *is commutative iff* ***`aggregate`***$(seq, comb, \perp, rdd)$ *is commutative.*
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SPARK `aggregateByKey` and `reduceByKey`

- work on PairRDDs: elements are (k, v)

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Discussion

Conditions for deterministic aggregation:

1. $(\mathit{img}(\mathtt{foldl}(\mathit{seq}, \perp)), \mathit{comb}, \perp)$ is a commutative monoid and
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- result for MapReduce [Chen, Hong, Sinha, Wang; TACAS'15]
  - $\mathbb{N}, +, \times$, control(loop-free): undecidable

Commutativity in SPARK aggregation

Case studies:

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- manual evaluation of SPARK ML library
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- found a redundancy in the SPARK Graph library

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Our contribution:

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- also extended to **`aggregate`** over graphs