

# First-Order Theories

## Exercises

**Exercise 1** Consider the formal model of first-order logic whose signature contains  $=/2$ , which is interpreted as the identity relation on the domain. Write a first-order theory for this model such that its interpretations are singleton sets.

**Exercise 2** Find an interpretation of Peano arithmetic ( $\mathcal{T}_{PA}$ ) such that its domain contains elements constructed using the following data structure:

```
struct T {
    struct T* next;
}
```

**Exercise 3** Find a nonstandard model of  $\mathcal{T}_{PA}$ .

**Exercise 4** Assume a theory  $\mathcal{T}$  with the signature  $\Sigma_{\mathcal{T}} = \{r/2\}$  where  $r$  is a predicate symbol and the following axioms:

1.  $\forall x. \neg r(x, x)$
2.  $\forall x, y. r(x, y) \rightarrow \neg r(y, x)$
3.  $\forall x, y, z. r(x, y) \wedge r(y, z) \rightarrow r(x, z)$

Find an interpretation of  $\mathcal{T}$ .