Efficient Inclusion Checking on Explicit and Semi-Symbolic Tree Automata

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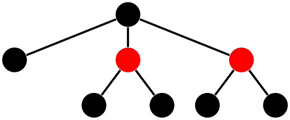
Outline

Trees

Very popular in computer science:

- data structures,
- computer network topologies,
- distributed protocols, . . .



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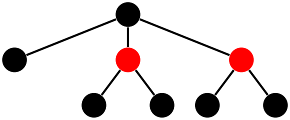

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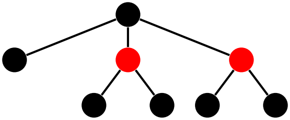

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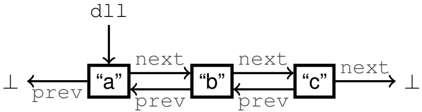

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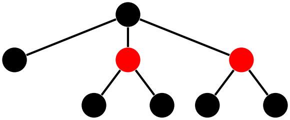

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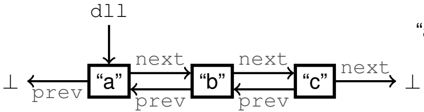

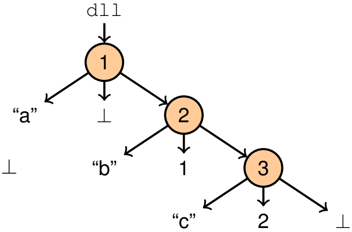

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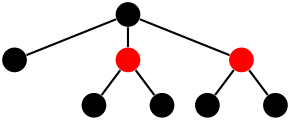

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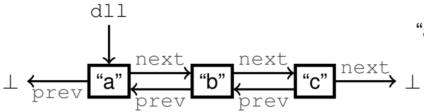

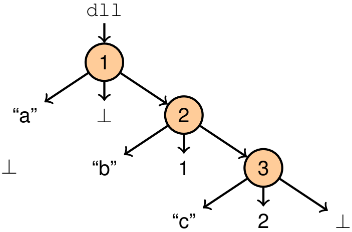

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Tree Automata

Finite Tree Automaton (TA): $\mathcal{A} = (Q, \Sigma, \Delta, F)$

- extension of finite automaton to trees:
  - Q ... set of states,
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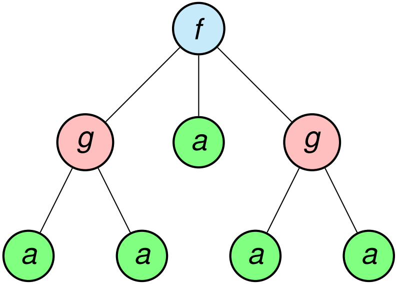

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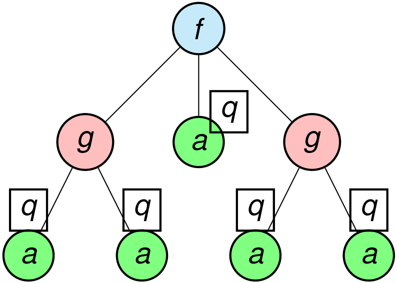

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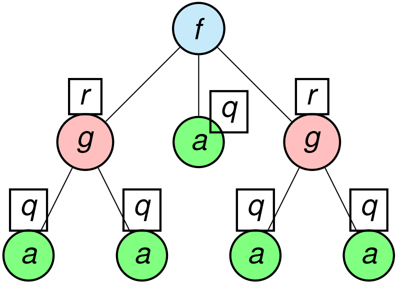

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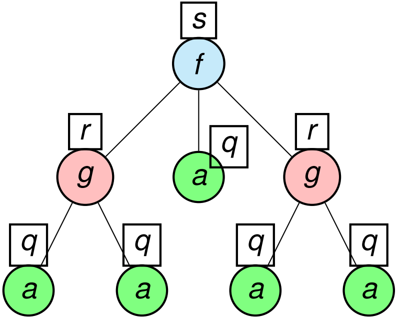

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- handling large alphabets (MSO, WSkS).

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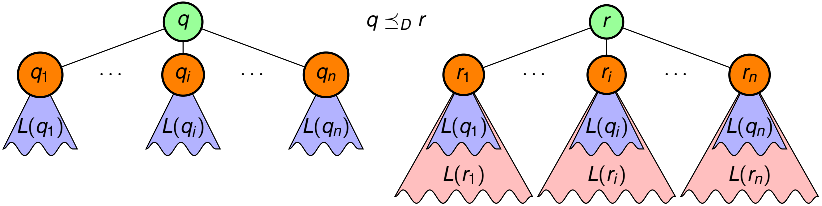

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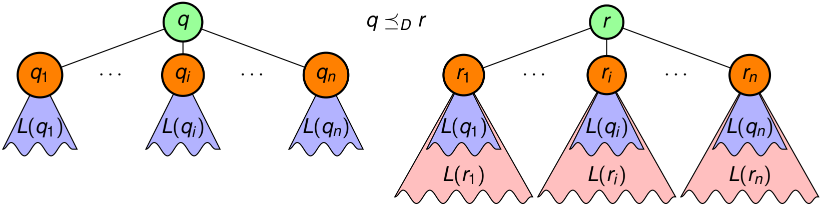

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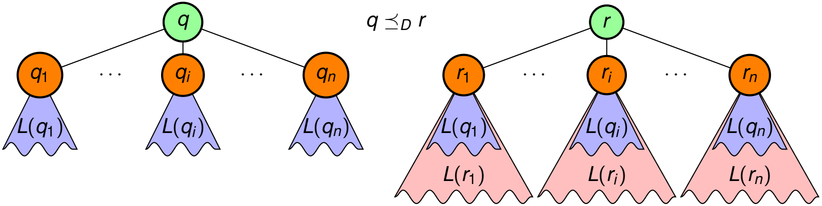

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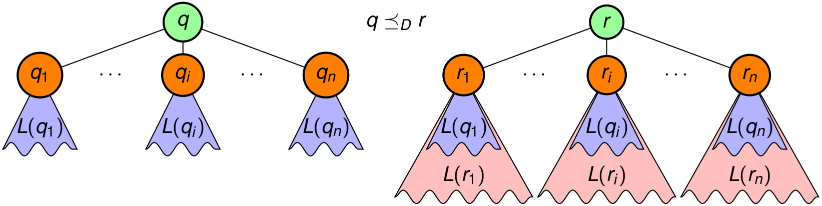

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- upward simulation
  - ▸ not compatible with language inclusion,
  - ▸ but can be used to speed up exact checking

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Exact: **EXPTIME-complete . . .**

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  - antichains[1]
  - antichains combined with simulation[2,3]

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[2] L. Doyen, J.-F. Raskin. Antichain Algorithms for Finite Automata. TACAS'10.

[3] P. Abdulla, Y.-F. Chen, L. Holík, R. Mayr, T. Vojnar. When Simulation Meets Antichains. TACAS'10.

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5. If you encounter (f, R), where $f \in F_S, R \cap F_B = \emptyset$, return false.
6. If no new pairs are found, return true.

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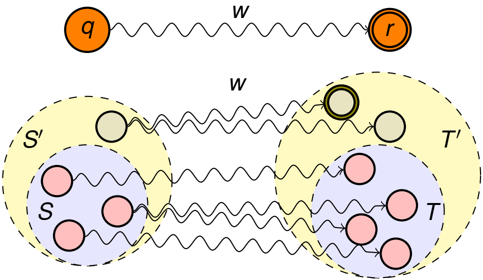

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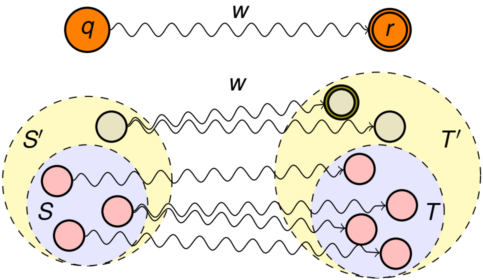

2. use simulation to further prune the searched space.

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- Upward simulation $\rightarrow$ hard to compute and too strong. ☹
- Not compatible with downward simulation (easy & rich). ☹

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- inspired by XML Schema containment checking[4],

[4] H. Hosoya, J. Vouillon, B. C. Pierce. Regular Expression Types for XML. ACM Trans. Program. Lang. Sys., 27, 2005.

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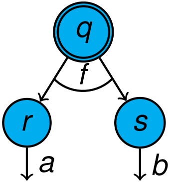

$\mathcal{A}_B$

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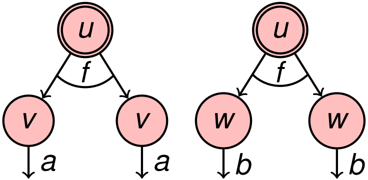

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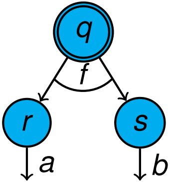

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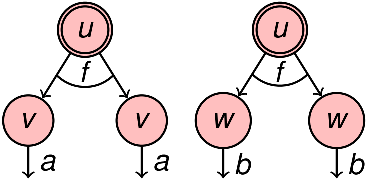

$\mathcal{L}(q) \subseteq \mathcal{L}(u)$ if and only if

$$\mathcal{L}(r) \times \mathcal{L}(s) \subseteq (\mathcal{L}(v) \times \mathcal{L}(v)) \cup (\mathcal{L}(w) \times \mathcal{L}(w))$$

(language inclusion of tuples!)

Checking language inclusion of tuples

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$$(\mathcal{L}(v_1) \times \mathcal{L}(v_2)) \cup (\mathcal{L}(w_1) \times \mathcal{L}(w_2)) \neq (\mathcal{L}(v_1) \cup \mathcal{L}(w_1)) \times (\mathcal{L}(v_2) \cup \mathcal{L}(w_2))$$

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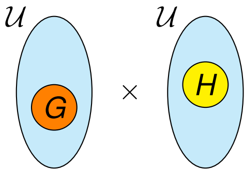

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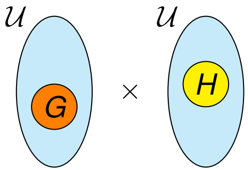

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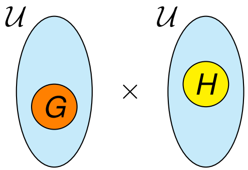

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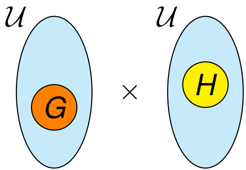

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Using distributive laws, this becomes

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$\Rightarrow$ **Checking language inclusion can be done component-wise.** $\Rightarrow$

$$\iff \begin{array}{cccc}((\mathcal{L}(r) \subseteq \mathcal{L}(\{v_1, w_1\})) & \vee & (\mathcal{L}(s) \subseteq T_\Sigma)) & \wedge \\ ((\mathcal{L}(r) \subseteq \mathcal{L}(v_1)) & \vee & (\mathcal{L}(s) \subseteq \mathcal{L}(w_2))) & \wedge \ldots\end{array}$$

Basic Downward Inclusion Checking Algorithm

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  - for all clauses …
  - exists a position such that inclusion holds.
- Sooner or later, the DFS either
  - reaches a leaf, or
  - reaches a pair (q_S, P_B) which is already in W.

Optimised Downward Inclusion Checking Algorithm

Optimisations:

1. It is possible to maintain a cache *NN* of pairs (q_S, P_B) for which $\mathcal{L}(q_S) \not\subseteq \mathcal{L}(P_B)$ has been shown and prune the search.

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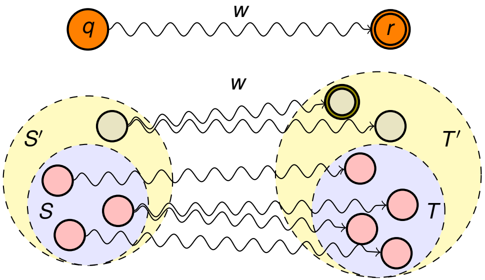

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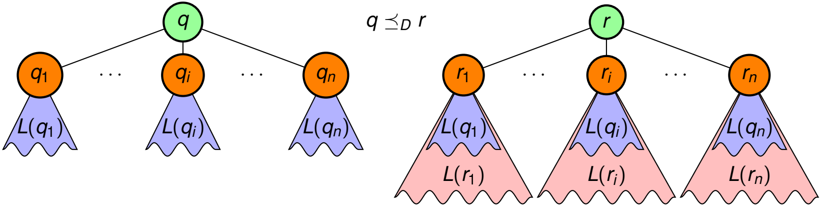

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4. Furthermore, workset can be also maintained w.r.t. $\preceq_D$.
5. Even further, if $\exists s \in S : q \preceq_D s$, then surely $\mathcal{L}(q) \subseteq \mathcal{L}(S)$.

Experiments

Size	50–250	400–600
Pairs	323	64
Timeout	20 s	60 s
`Up`	31.21 %	9.38 %
`Up+s`	0.00 %	0.00 %
`Down`	53.50 %	39.06 %
`Down+s`	15.29 %	51.56 %
Avg up	1.71	0.34
Avg down	3.55	46.56

a)

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`Down+s`	18.18 %	79.69 %
Avg up	1.33	9.92
Avg down	3.60	2116.29

b)

a) Comparison of methods (w/ simulation computation time).

b) Comparison of methods (w/o simulation computation time).

Semi-Symbolic TA

Several FV approaches yield automata with large alphabets:

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- decision procedures of some logics: WSkS, MSO.

Current approach:

- use the MONA tree automata package (MTBDD-based)
- But only deterministic automata supported $\rightarrow$
  - often runs out of reasonable memory or time.

Dual representation

- Multi-terminal binary decision diagrams (MTBDDs)

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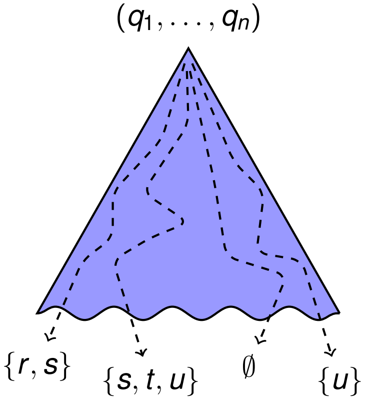
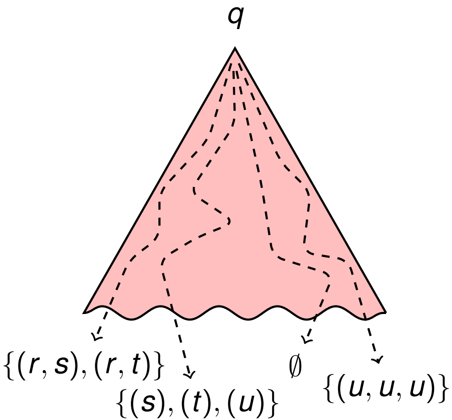
- Bottom-up:



- Top-down:



Bottom-up : inspired by MONA, but has sets of states in leaves.

Top-down : sets of state tuples in leaves.

Semi-Symbolic Encoding of Non-Deterministic TA

Algorithms for

- union,
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    - based on M. Henzinger, T. Henzinger, and P. Kopke's algorithm.

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Experiments:

- Use of CUDD to implement MTBDDs.
- $\sim$ 8500 times faster downward inclusion checking than explicit representation for tested automata with large alphabets.

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- ...that makes use of antichains and downward simulation.
- A new symbolic encoding of non-deterministic tree automata proposed.

Future work

- Optimise the downward inclusion to also cache pairs (q, S), such that $\mathcal{L}(q) \subseteq \mathcal{L}(S)$.

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- Replace CUDD with a more efficient MTBDD package.
- Improve the symbolic downward simulation algorithm.
- Create a tree automata package replacing MONA.

Thank you for your attention.

Questions?